

What Makes Music Sound Good?

Intuitively, some combinations of notes sound better than others. What's the difference between good-sounding and bad-sounding combinations? While we cannot answer this question absolutely (since there's no accounting for taste), we can identify five different properties that are common to a very wide range of styles, from early Medieval music to contemporary popular music. They are:

1. *Conjunct melodic motion*. Melodies tend to move by short distances from note to note. Large leaps sound inherently unmelodic.
2. *Harmonic consistency*. The chords in a passage of music, whatever they may be, tend to be structurally similar to one another.
3. *Acoustic consonance*. Some chords sound intrinsically good or pleasing. These are said to be *consonant*.
4. *Scales*. Over small spans of musical time (say 30 seconds or so), most musical styles tend to use just a few types of notes, between 5 to 8.
5. *Centricity*. Over moderate spans of musical time, one *tonic* note is heard as being more prominent than the others, appearing more frequently and serving as a goal of musical motion.

These five properties make an enormous difference to our immediate experience; in fact, you can take completely random notes and make them sound reasonably musical, simply by forcing them to conform to these requirements.

In this class, we'll think about two different sorts of questions. First, how *in principle* can these various properties be combined? For example, if you want to combine the first two properties (short melodies and chords that sound similar) what sorts of chords should you use? And second, how have previous composers used these different features? We will see that different styles (including Renaissance music, classical music, Romantic music, jazz and rock) all involve slightly different ways of deploying the same basic properties. Thus when you dig deep enough, it turns out that all these different styles exhibit the same basic structure.

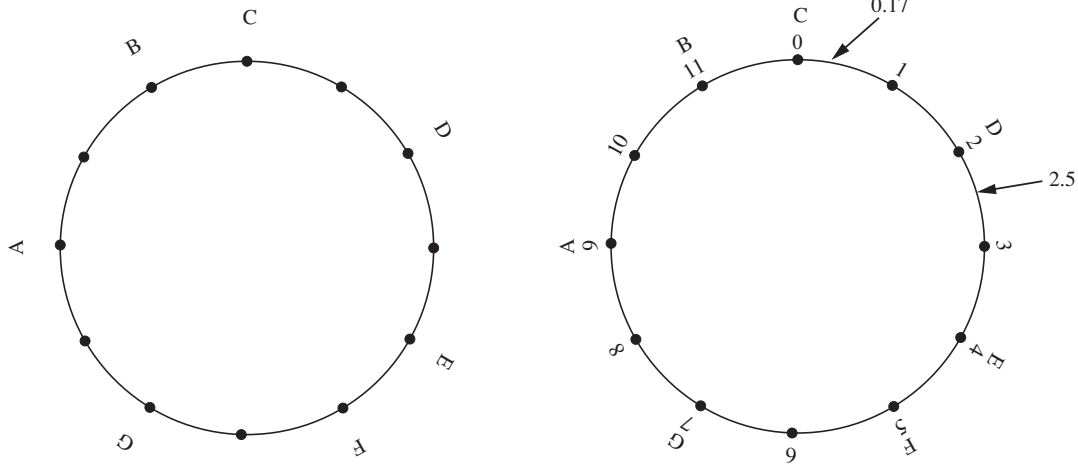
There's a reason for this: the five properties constrain each other in interesting ways. Intuitively, you might think that the five constraints could be satisfied in unimaginably many ways, just as you might intuitively think that the universe was filled with habitable planets. But in fact the situation is more interesting than that: the procedures of Western music are much closer to being unique solutions to the basic musical problem of combining the five properties. (Similarly, it turns out that the universe may not contain

so many planets that can sustain life, much to the disappointment of science fiction authors.) For instance, if you want to combine conjunct melodic motion with harmonic consistency, it turns out you need to use the acoustically consonant chords of traditional Western music: major triads, dominant seventh chords, and so on.

For essentially similar reasons, much Western music has a kind of hierarchical self-similarity, with the behavior of scales mirroring that of chords. In many Western styles, it is common to switch scales—for instance, the opening section of a piece might use the white notes C-D-E-F-G-A-B while the next section uses the G major scale G-A-B-C-D-E-F $\sharp$. We can understand this process by saying that the note F *moves up* to F $\sharp$ by a short distance. This is basically the same thing that happens when one chord moves to another, though now on the level of the scale rather than the chord.

The ultimate purpose of this course is twofold. First, I want you to develop fluency in a range of styles from medieval music to contemporary rock. The point here is to have a sense for all the different ways in which Western composers make music, and not get too “locked in” to any one way of doing things. You might come to this class fluent only in classical music, or jazz, or rock, but I want you to leave with a reasonable understanding of all three styles. Second, I want to understand the deeper similarities between these superficially different-sounding musics—the “hidden roads” them. Ultimately, it’s not enough just being able to imitate a bunch of different musical procedures, like a robot executing a series of pre-programmed dance steps. I’d also like you to understand *why* these practices are the way they are.

Fundamentals

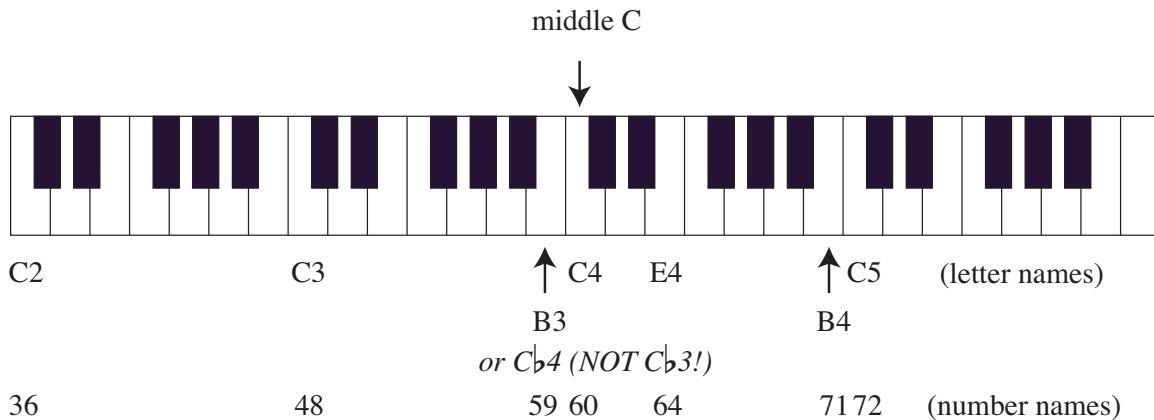


1. Pitch and pitch class. A *pitch* is a specific note, like middle C or the G a perfect fifth above it. A *pitch class* is a note type, like “C” or “G”—it’s what we end up with when we ignore what octave pitches are in. Pitches live on a line, while pitch classes live on a circle. (Pitch is like time, whereas pitch class is like time-of-day.) Note Western music divides the pitch-class circle into 12 units, just like an ordinary clock, meaning you’re never far from a model of pitch-class space.

2. Naming pitch classes. We have two systems for naming pitch classes: letters and numbers. Letters came first, and only the white notes have their own letter names. The black notes (like C $\sharp$) have to be named relative to some other pitch class. Note that every pitch class has an infinite number of letter names: C, B $\sharp$, A $\sharp\sharp\sharp$, etc. By contrast, in the numerical system every pitch class has a unique name (C = 0, C $\sharp$ /D $\flat$ = 1, D = 2, etc.). We can assign names even to pitch classes not found on the ordinary piano keyboard (for instance, the pitch class 2.5 is halfway between D and E $\flat$.)

3. Naming pitches. We can use a similar system to label pitches: the most common method, *scientific pitch notation*, combines a letter name with a number indicating what octave a note is in; middle C is given the octave number 4, as are all notes less than an octave above it. **NB:** accidentals are applied *after* the letter and number are combined to generate a pitch: the note just below middle C can be labeled either B3 or C $\flat$ 4! Similarly, C4 can also be called B $\sharp$ 3! This is terribly confusing but there it is.

It is also possible to use numbers to label pitches. Here middle C is given the number 60, the C $\sharp$ above that is 61, the D above that is 62, and so forth. To go from pitch numbers to pitch class numbers divide by 12 and keep only the remainder.



4. Naming intervals. An *interval* is a musical distance. Intervals can be named relative to letter names and relative to pitch numbers. Numerical pitch labels are the simplest, but they are less commonly used: the interval between middle C (C4, pitch number 60) and the D immediately above it (D4, pitch number 62) is *two ascending semitones*. The interval between D4 (62) and G3 (55) below it is *seven descending semitones*. Intervals in this sense are sometimes called “chromatic intervals” or “chromatic semitones.” To find the interval between two notes just count keys on a keyboard, or frets on a guitar, or ...

Unfortunately, the most common way to talk about musical distance (intervals) is a letter-based system. *Strictly speaking, this system is defined only for notated music.* We cannot identify the interval between two notes just by listening. However, there are a set of common conventions for notating music that typically make it possible to guess the correct letter-name interval, merely by listening.

The interval between interval between any C (such as C $\sharp$ 4 or C $\flat$ 4) and any D immediately above it (such as D $\sharp$ 4) is called an ascending *second*. Believe it or not, this is because C and D are *one* letter name apart. Similarly, the interval between any C and any form of the E immediately above that is called an ascending *third*—again, because they are *two* letter notes apart. Note that a second is not necessarily smaller than a third: the interval C–D $\sharp$ is an *augmented second*, which is larger than the interval C $\sharp$ –E $\flat$, a *diminished third*. See the table at the end of this handout.

5. Registral inversion. The (registral) inversion (or complement) of an interval is the interval formed by transposing the top note down by octave until it is the bottom note. For instance, C4–E4 is a major third; transposing the E down an octave, we get its inversion, E3–C4, a minor sixth. An interval and its registral inversion always make up an octave. Inversion transforms major into minor and perfect into perfect: thus the minor third becomes the major sixth and the perfect fifth becomes the perfect fourth.

Letter Name Distance	Chromatic Distance	Name	Abbreviation
0 (e.g. C-C) <i>unison</i>	-1 semitones (C-C $\flat$)	diminished unison	d1
	0 semitones (C-C)	unison (or perfect unison)	P1
	1 semitone (C-C $\sharp$)	augmented unison	A1
1 (e.g. C-D) <i>second</i>	0 semitones (C $\sharp$ -D $\flat$)	diminished second	d2
	1 semitone (C $\sharp$ -D)	minor second	m2
	2 semitones (C-D)	major second	M2
	3 semitones (C-D $\sharp$)	augmented second	A2
2 (e.g. C-E) <i>third</i>	2 semitones (C $\sharp$ -E $\flat$)	diminished third	d3
	3 semitones (C $\sharp$ -E)	minor third	m3
	4 semitones (C-E)	major third	M3
	5 semitones (C-E $\sharp$)	augmented third	A3
3 (e.g. C-F) <i>fourth</i>	4 semitones (C-F $\flat$)	diminished fourth	d4
	5 semitones (C-F)	perfect fourth	P4
	6 semitones (C-F $\sharp$)	augmented fourth (<i>tritone</i>)	A4
4 (e.g. C-G) <i>fifth</i>	6 semitones (C-G $\flat$)	diminished fifth (<i>tritone</i>)	d5
	7 semitones (C-G)	perfect fifth	P5
	8 semitones (C-G $\sharp$)	augmented fifth	A5
5 (e.g. C-A) <i>sixth</i>	7 semitones (C $\sharp$ -A $\flat$)	diminished sixth	d6
	8 semitones (C-A $\flat$)	minor sixth	m6
	9 semitones (C-A)	major sixth	M6
	10 semitones (C-A $\sharp$)	augmented sixth	A6
6 (e.g. C-B) <i>seventh</i>	9 semitones (C $\sharp$ -B $\flat$)	diminished seventh	d7
	10 semitones (C-B $\flat$)	minor seventh	m7
	11 semitones (C-B)	major seventh	M7
	12 semitones (C-B $\sharp$)	augmented seventh	A7
7 (e.g. C-C, an octave above) <i>octave</i>	11 semitones (C-C $\flat$)	diminished octave	d8
	12 semitones (C-C)	octave (or perfect octave)	P8
	13 semitones (C-C $\sharp$)	augmented octave	A8

Additional Rules

1. an interval one chromatic semitone smaller than a diminished interval is “doubly diminished”; an interval one semitone larger than that is “triply diminished,” etc.

2. an interval one chromatic semitone larger than an augmented interval is “doubly augmented”; an interval one semitone larger than that is “triply augmented,” etc.

3. after the octave comes the ninth, tenth, eleventh, and so forth. So the interval from C $_4$ to E $\flat_5$ is a minor tenth.

I’ve used ascending intervals here; descending intervals work the same way.

6. Bonus fact. One can use the same system to identify rhythms. For instance, we can label the beats in a piece using numbers, and we can use a circle to represent the beats in a repeating measure.

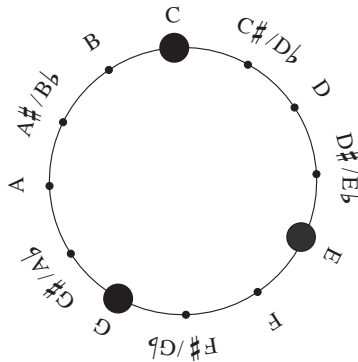
The top staff is in 12/8 time. It shows a sequence of notes: a dotted quarter note, a dotted quarter note, a dotted quarter note, a dotted quarter note, a dotted quarter note, a dotted quarter note, a dotted quarter note, a dotted quarter note, a dotted quarter note, a dotted quarter note, a dotted quarter note, and a dotted quarter note. The numerical labels below the staff are: 0, 3, 6, 10, 12, 15, 18, 22, 0, 3, 6, 10. Below these labels is the text "or: 0 3 6 10 0 3 6 10".

The bottom staff shows a sequence of notes: a quarter note, a quarter note, a quarter note, a quarter note, a quarter note, a quarter note, a quarter note, a quarter note, a quarter note, a quarter note, a quarter note, and a quarter note. The numerical labels below the staff are: 0, 2, 4, 5, 7, 9, 11, 0, 2, 4, 5, 7, 9, 11.

If you do this, you discover some interesting things. For instance, the major scale, 0-2-4-5-7-9-11 in numerical pitch-class notation, is a very common rhythm in Africa! (Why is this?) Musically, this means that we can translate various compositional ideas between the pitch domain and the rhythmic domain—for example, composers like Steve Reich use analogues to traditional modulation!

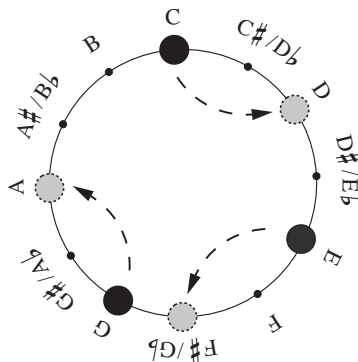
Chords, chord-types, and Counterpoint

1. Chord. Ordinary terms such as “C major chord” refer to pitch classes rather than pitches. When we say that a collection of notes is a C major chord we mean that it contains the notes C, E, and G in any octave. Sometimes these notes might be played at the same time, and sometimes they might be played one right after each other. In the latter case, it doesn’t matter which note comes first. It follows that we can model chords as an *unordered collection of pitch classes*, represented geometrically by an unordered set of points on the circle. For instance, the C major chord is the set {C, E, G} represented by the following points:



2. Chord types and transposition. To *transpose* a passage of music is to move every note by the same distance. Transposition is important because it *preserves the structure* of a chord, or in other words the distances between its notes. This means that transpositionally related chords sound similar. (In fact, most musicians cannot tell a C major chord, played on Monday, from a D major chord played on Tuesday—people who can do this have *absolute pitch*; the rest of us are stuck with *relative pitch*.)

Transposition is represented geometrically by rotation in the pitch class circle, as illustrated by the following:

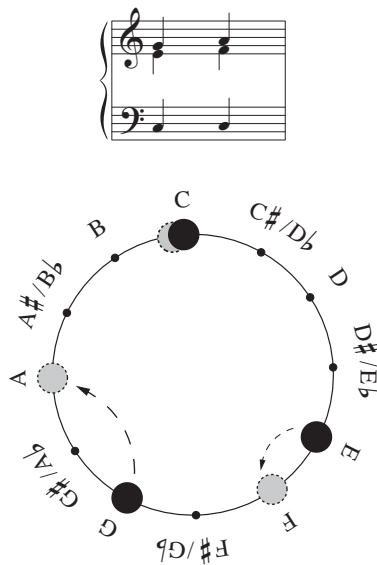


All major chords are related by rotation/transposition; all minor chords are related by rotation/transposition; all dominant sevenths are related by rotation/transposition; etc. We can say that two chords belong to the same *chord type* if they are rotationally related.

3. Counterpoint. Counterpoint is the art of combining multiple melodies to form a series of meaningful chords. It is a central ingredient of Western music.

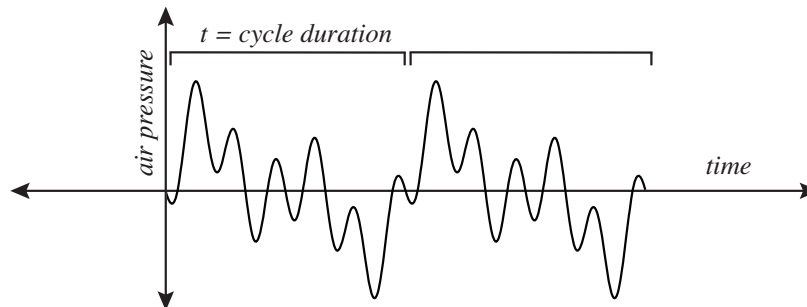
Suppose you would like to write music that uses a collection of similar sounding chords. (For our purposes, we can take “similar sounding” to mean “transpositionally related.”) This means you want to use chords that are related by rotation on the circle. But you also want to connect the chords so that no chord moves by very much, so that the transition can be represented as a series of structurally similar melodies. In other words, you want to find a pair of chords that are related in two ways: first, by rotation on the pitch class circle (so that they sound similar), and second by a short-distance mapping that moves the notes of the first chord to those of the second?

How is this possible? The answer is that this works best if we have a chord that divides the octave nearly evenly. This means that the notes of the first chord never have to move far to get to the second.



Basic Musical Acoustics

1. Sound. Sound consists in small fluctuations of air pressure, akin to the changes in barometric pressure familiar from the weather report. These fluctuations are heard to have a definite pitch when they repeat themselves (at least approximately) after some period of time t .



The reciprocal of the period, $1/t$, is the *fundamental frequency* of the sound—a number that measures how many repetitions of the pattern occur per unit of time. (Fundamental frequencies are labeled in *cycles per second* or *Hertz*.) Most musicians, lacking absolute pitch, are not sensitive to the absolute frequency of individual sounds: instead, they are sensitive to the ratios between fundamental frequencies. Suppose, on Tuesday, someone whistles a tune whose successive pitches have frequencies $f, g, h, \dots$. On Wednesday, if asked to reproduce the same tune, she is likely to whistle notes whose frequencies are $cf, cg, ch, \dots$, where c is some arbitrary number (typically close to 1). Musicians say that the whistler has *transposed* the tune: she has changed each of its fundamental frequencies in a way that preserves the ratios between them. This suggests that the ratio represents a kind of musical distance, and that listeners are attuned to the distances between notes rather than their absolute positions in frequency space.

It turns out that the ratio 2:1 corresponds to the octave. (If a note has frequency f , then $2f$ is the note an octave higher.) The ratio 3:2 is the perfect fifth, 4:3 is the perfect fourth, 5:4 is the major third, and 6:5 is the minor third. It is actually somewhat awkward to work with ratios and fundamental frequencies: multiplication and division can be confusing, and 110, 220, and 440 **but not 330** all represent the note A ... For that reason, musicians often use the numbering system we learned about in the last class.

2. The origins of consonance and dissonance. Certain chords, such as perfect fifths and major triads, sound stable and restful, or *consonant*, while other sonorities, such as the chromatic cluster $\{C, C\sharp, D\}$, sound unstable and harsh, or *dissonant*. Most Western styles exploit this difference: consonant sonorities tend to appear as musical destinations or points of rest, while dissonant sonorities tend to be more active and unstable. Dissonance is the spice in the musical stew.

The eighteenth-century mathematician Jean-Baptiste Fourier showed that any periodic waveform can be represented as the sum of sine waves whose frequencies are integer multiples of its fundamental frequency f .

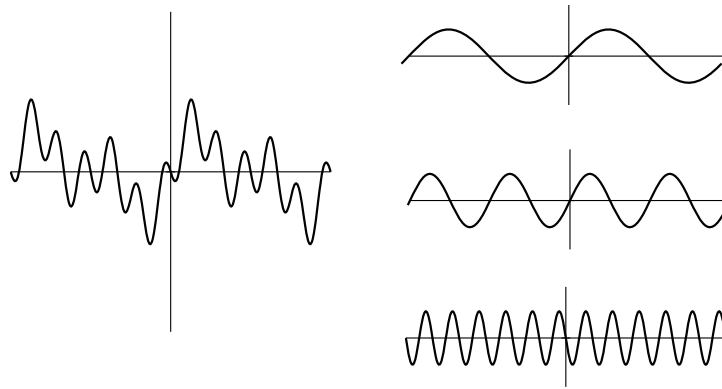


Figure 1. The periodic sound on the left has frequency f . (Two repetitions of the wave are shown.) It can be analyzed as the sum of the sine waves on the right, with periods f , $2f$, and $6f$. (These repeat twice, four times, and twelve times in the same horizontal space.)

When the periodic function represents a musical sound, these waves are called *overtones*, *partials*, or *harmonics*.

The nineteenth-century German psychologist Hermann von Helmholtz proposed that two pitches are heard to be consonant if their overtones are aligned, or else are sufficiently far apart from each other so as not to interfere. Dissonance is caused when overtones are close together, but not perfectly aligned; this creates a sense of “roughness” or harshness. (Twentieth-century auditory physiologists have shown that the ear’s basilar membrane decomposes incoming soundwaves into overtones, more-or-less as Fourier described.) Though there is disagreement about just how complete this story is, most contemporary psychologists believe that it is at least partly right.

Most Western instruments produce “harmonic” sounds that, when analyzed as Fourier described, have relatively strong lower overtones f , $2f$, $3f$, $4f$. The overtones of several of these sounds will match when their fundamental frequencies are related by simple whole-number ratios. Consequently, as Pythagoras discovered roughly 2700 years ago, pairs of harmonic tones will be particularly consonant when their fundamental

frequencies are approximately in simple whole-number ratios such as 3:2 or 5:4. Groups of tones will be consonant when their fundamental frequencies are all mutually in simple integer ratios: chords such as 3:2, 5:4:3, 7:6:5:4, 9:8:7:6:5 and so on. **However, if our instruments produced different overtones, then we would hear different intervals as being consonant!**

3. On the piano, the consonant chords divide the octave nearly but not perfectly evenly. Here are some of the most consonant chords, of sizes two through seven. All divide the octave nearly-but-not quite exactly evenly.

Notes	Name
{C, G}	Perfect fifth
{C, E, G}	Major triad
{C, E $\flat$, G}	Minor triad
{C, E, G, A}	Major triad “add six”
{C, E, G, B $\flat$ }	Dominant seventh chord
{C, E, G, B}	Minor seventh chord
{C, D, E, G, A}	Pentatonic scale
{C, E, G, B $\flat$, D}	Dominant ninth chord
{C, D, E, F, G, A}	Diatonic hexachord
{C, E, G, B $\flat$, D, F $\sharp$ }	Dominant “sharp eleven”
{C, D, E, F, G, A, B}	Diatonic scale
{C, D, E, F $\sharp$, G, A, B $\flat$ }	Melodic minor ascending

The mathematical reason for this is slightly complicated. It’s related to the fact that, when we think in terms of fundamental frequencies, the perfect fifth and the major triad divide the octave exactly evenly: the note 330 Hz (E4) divides the octave between 220 Hz (A3) and 440 Hz (A4) into two equal (110-Hz-sized) parts. (Note that E4 is a perfect fifth above A4.) Similarly, the A major triad 330 Hz (E4), 440 Hz (A4), and 550 Hz (C $\sharp$ 5) divides the octave between 330 Hz (E4) and 660 Hz (E5) into three equal (110-Hz-sized pieces). It turns out that when we go from fundamental frequencies to ordinary note labels, we transform perfectly even divisions into nearly even divisions.

4. The deep fact. The basic sonorities of Western music, such as perfect fifths, major triads, dominant seventh chords, and so on, turn out to be important for two distinct reasons. Considered as individual sonic objects, they are acoustically consonant, and hence sound pleasing in their own right. But since these chords divide the piano-keyboard octave nearly evenly, they are also ideal for combining harmonic consistency with conjunct melodic motion—as discussed in an earlier lecture!

5. Bonus mathematical fact. As we learned last time, we can name pitches by labeling the keys of the piano. Here A3 (= 220 Hz) is 57, A4 (= 440 Hz) is 69, A5 (= 880 Hz) is 81, and so on. With fundamental frequencies we *multiply by two* to ascend by octave; with numerical pitch notation, we *add twelve*. To convert from a multiplication-based system to an addition based system, we use logarithms. In this case, the equation is:

$$p = 69 + 12\log_2 (f/440)$$

or to go the other way

$$f = 440 \times 2^{(p - 69)/12}$$

Here, f is a note's fundamental frequency, and p is the numerical pitch label. In general, addition and subtraction are easier than multiplication and division, which is why musicians prefer numerical pitch labels to fundamental frequencies.

STYLE 1: SIMPLE MEDIEVAL COUNTERPOINT

Our first musical style is modeled after the earliest examples of Western counterpoint: the two voice 11th-century examples found in the text *Ad organum faciendum* (“How to make counterpoint”). The instructions below encapsulate (more or less!) the cutting-edge radical music that was being made 1000 years ago!

I. Pitch. The only notes allowed are the seven white notes and B $\flat$.

II. Rhythm. The music should be written in a steady, unchanging rhythm, without rests. You can notate it in either of two ways: (a) on two staves using whole notes for each voice; or (b), following the examples below, on one staff with one voice in whole notes and the other using filled-in noteheads. The music should be divided into phrases, of approximately 5-10 notes. This is to allow for a slight pause for breath at the end of each phrase. Draw a bar line between phrases.

III. Voices and ranges. Write for two voices singing simultaneously, in precisely the same steady, unchanging rhythm. Observe the following ranges:

Soprano: C4 $\leftrightarrow$ G5 Alto: G3 $\leftrightarrow$ C5 Tenor: C3 $\leftrightarrow$ G4 Bass: G2 $\leftrightarrow$ C4

You can write for any pair of voices except soprano and bass. (In particular, you can write for, e.g., two sopranos.) In general, voices should be less than an octave apart. *They should never be more than an octave and a fifth apart.*

IV. Counterpoint. Observe the following rules:

1. Each voice should move mostly by step.
2. Leaps of more than a fifth are not allowed.
3. Consecutive leaps in the same direction must both be thirds; no more than two in a row.
4. Voices cannot move by tritone.
5. Parallel octaves and unisons should be used only very occasionally, and there should never be more than one parallel octave (or unison) consecutively.
6. You should try to make sure your two voices are reasonably independent: that is, they should not move in parallel for more than three consecutive notes.
7. Voices should cross only occasionally
8. Each voice should make a good melody, with a clear high point, delineating a specific register of no more than an octave, etc.

V. Harmony. Intervals are grouped into three classes: perfect consonances, imperfect consonances, and dissonances. Phrases must begin with a perfect consonance and end with either a unison or an octave. Imperfect consonances are permitted only in the middle of the phrase. Imperfect consonances should be used somewhat sparingly, however: they were thought to be somewhat-dissonant sounds that add “spice” to the more stable perfect consonances.

Dissonances are absolutely forbidden at all times.

unison (or octave), perfect fourth (or eleventh), perfect fifth (or twelfth)	perfect consonance
third (or tenth), sixth	imperfect consonance
second, seventh, augmented fourth (or eleventh), diminished fifth (or twelfth)	dissonance

VI. Cadences. Each small phrase of music should end with the two voices moving by step, in contrary motion, to a unison or octave. Example: tenor moves from B3 to C4, while alto moves from D4 to C4. Or: tenor moves from D4 to C4 while alto moves from B4 to C5.

VII. Cross relations: B and B $\flat$. You can use the note B $\flat$ to avoid the tritone between F and B. Be careful about the tritone between E and B $\flat$, however! You should never have two consecutive chords, one with a B $\sharp$ and the other with a B $\flat$. This is called a “cross relation.”

VIII. Three-voice counterpoint. It is also possible to use these rules to write three voice counterpoint. The dissonance rules in section V apply to all three *pairs* of voices—thus the piece has to begin and end with three-voice sonorities using only perfect consonances; imperfect consonances are acceptable only in the middle of the phrase. At the cadence, the outer voices should obey the rules in section VI, while the middle voice moves by step or else is held constant. The middle voice should not cross above the top voice or below the bottom voice.

Try to avoid moving all three voices in the same direction.

Organum: Alleluia Justus ut palma (ca. 1100)

[Soloists] [Choir]

Al-le - lu - ia Al-le - lu - ia.

[Soloists]

Ius - tus ut pal - ma flo - re - bit

et si - cut ce

[Choir]

- drus mul - ti pli - ca - bi - tur.

Milan, Biblioteca Ambrosiana, MS M.17 sup., ed. Hans Heinrich Eggebrecht and Frieder Zaminer, *Ad organum facien- dum, Lehrschriften der Mehrstimmigkeit in nachguidonischer Zeit* (Mainz, 1970), p. 53.
© copyright 1970 by B. Schotts Söhne, Mainz. Used by permission of European American Music Distributors Corp. Sole U.S. Agent for B. Schotts Söhne.

Alleluia Justus ut palma florebit,
et sicut cedrus multiplicabitur.

Alleluia. The righteous shall flourish
like a palm tree and shall multiply like a cedar.

a. Cunctipotens genitor

11th century

Cun-cti-po-tens ge-ni-tor de-us, om-ni-cre-a-tor, e- ley-son. Chri-ste De-i splendor, vir-tus pa-tris-

que so-phi-a, e- ley-son. Am-bo-rum sa-crum spi-ra-men, ne-xus a-mor-que, e- ley-son

Cun-cti-po-tens ge-ni-tor De-us o-mni cre-a-tor e-

ley-son. Chri-ste De-i splen-dor, vir-tus pa-tris-que

so-phi-a, e- ley-son. Am-bo-rum sa-crum spi-ra-men

nex-us a-mor-que, e- ley-son.

SCALES, SCALES, SCALES

I. Scales and musical distance. Abstractly, a scale determines a unit of musical distance, the *scale step*. In the scale C-D-E-F-G-A-B-C, the note D is one scale step above C. The note E is two scale steps, or a *third*, above C. F is three scale steps, or a *fourth*, above C. And so on. We can refer to this sort of distance, measured along the scale, as *scalar distance*.

We can transform any scalar passage by shifting each of its notes by the same number of scale steps. This process is called *scalar transposition* or transposition within the scale. For instance, we transform C-D-E (“Do, a deer”) into D-E-F (“Re, a drop”) by moving each note in the first pattern up one scale step. Scalar transposition introduces subtle but important variations into music: D-E-F sounds audibly similar to C-D-E, even though D-F is a minor third while C-E is a major third. Musically, it is important that these variations not be too large; otherwise scalar transposition will distort the character of a musical passage beyond recognition. The variations will be small only when a scale divides the octave relatively evenly.

Chromatic transposition is transposition relative to the chromatic scale. Usually, when a musician says “transposition,” she has chromatic transposition in mind.

NOTE: scales give us a third way to measure musical distance, distinct from both the letter-name system and the numerical system. Consider the three ways of measuring the distance between the steps in the “octatonic” scale C-C $\sharp$ -D $\sharp$ -E-F $\sharp$ -G-A-B $\flat$.

Scale step	Scalar distance	Chromatic distance	Letter name distance
C-C $\sharp$	1	1	augmented unison
C $\sharp$ -D $\sharp$	1	2	major second
D $\sharp$ -E	1	1	minor second
E-F $\sharp$	1	2	major second
F $\sharp$ -G	1	1	minor second
G-A	1	2	major second
A-B $\flat$	1	1	minor second
B $\flat$ -C	1	2	major second

Here the distance C-C $\sharp$ is one scale step, but it is written as an augmented unison; by contrast, the distance D $\sharp$ -E is one scale step, but it is written as a minor second. There is no way to respell the notes so as to remove all the inconsistencies. In fact, unless a scale has seven notes, it is impossible to write the scale so that letter-name distance and scalar distance are consistent. This is one reason to be wary of standard musical notation!

II. Scale and mode, scale type and mode type. If we think of scales simply as devices for measuring musical distance, then scales do not need to have “first” notes. From this point of view, we could define a scale as a circular arrangement of pitch classes, such as C-D-E-F-G-A-B-[C]. (The brackets around the last C indicate that we have a circular ordering that has returned to its starting point.) A *mode* is a scale in which a particular note has been selected as primary, or *tonic*. Typically, the tonic note serves as the goal of melodic and harmonic motion; one typically finds that melodies end on the tonic of the mode. Notes of a mode are sometimes called *scale degrees* and are usually numbered starting from the tonic; the “first scale degree” is the tonic, the “second scale degree” is the note just above the tonic, and so on. For example, the C ionian mode C-D-E-F-G-A-B-C has C as its tonic and D as its second scale degree; the D dorian mode D-E-F-G-A-B-C-D has D as its tonic and E as its second scale degree. These two modes are different, even though they use the same notes!

The C diatonic scale C-D-E-F-G-A-B-[C] is different from the G diatonic scale G-A-B-C-D-E-F $\sharp$ -[G], since the former has an F $\natural$ while the latter has an F $\sharp$. But the two scales are related by chromatic transposition: every note in the G major scale is seven semitones above the corresponding note in the C major scale. For this reason, they sound very similar. Two scales belong to the same *scale type* if they are related by chromatic transposition. The C and G diatonic scales both belong to the *diatonic scale type*. Similarly, two modes to the same *mode type* if they are related by chromatic transposition; for example the D dorian mode D-E-F-G-A-B-C-D and the G dorian mode G-A-B $\flat$ -C-D-E-F-G belong to the *dorian mode type*.

WARNING: Musicians are not always consistent in their use of terminology: sometimes they use “scale” to mean “mode” (as in “the A natural minor scale”), and sometimes they use it to mean “scale type” (as in “the diatonic scale”). Sometimes “mode” means “mode type” (“the dorian mode”). As long as context makes your meaning clear, and it is perfectly fine to write and speak in this (somewhat informal) way.

III. Deriving the diatonic scale. Historically, the diatonic scale came *before* the chromatic scale. Where did the diatonic scale come from? The basic answer is that the diatonic scale is maximally saturated with consonant intervals (such as the perfect fifth, major third, and minor third) while also dividing the octave nearly evenly and containing a reasonably small number of notes.

Suppose we want to invent a scale that is (1) nearly even; (2) has only 5-8 notes; and (3) contains as many consonant intervals as possible. Since the perfect fifth is the most consonant interval, we might try building a scale so that every note has a perfect fifth above it: so C is in the scale, then we would include G, and D above that, and so on. Unfortunately, a stack of acoustically perfect fifths will *never* return to its starting point.

(Remember that the acoustically pure fifth is slightly different from the equal tempered fifth of the ordinary piano.) This means we need to cheat: one strategy is to use a stack of fifths C-G-D-A-E... where the last note is *almost* (but not quite) a fifth away from the first note. There turn out to be three salient possibilities: the five-note *pentatonic scale* C-G-D-A-E, whose last note is 8 semitones below the first; the seven-note *diatonic scale* F-C-G-D-A-E-B- whose last note is 6 semitones below the first; and the twelve-note *chromatic scale*, B $\flat$ -F-C-G-D-A-E-B-F $\sharp$ -C $\sharp$ -G $\sharp$ -D $\sharp$ whose “wolf fifth” D $\sharp$ -B $\flat$ is about $\frac{1}{4}$ of a semitone smaller than the others. Contemporary equal temperament cheats in another way, altering the fifths in the chromatic scale so that they are all exactly equal.

These three scales all divide the octave nearly evenly while being maximally saturated with perfect fifths. However, the diatonic scale has two advantages: it has fewer notes than the chromatic, while containing more thirds than the pentatonic. In fact, every note in the diatonic scale has either a major or minor third above it.

semitones: 5 5 5 5 5 5 6

4 3 4 3 3 4 3

2 2 1 2 2 2 1

Here, we see that if we take every third note of the diatonic scale, we get a stack of perfect fourths, with only one augmented fourth. If we take every other note of the scale we get a stack of major and minor thirds.

V. Deriving other familiar scales. There are a number of other scales that are important in Western (and other) music. All of these are saturated with consonant intervals, while also dividing the octave reasonably evenly.

For example, the *pentatonic scale* contains as many perfect fifths as a five-note scale can have (which is to say, four), while also dividing the octave reasonably evenly. The pentatonic scale has just two sizes of step (2 and 3 semitones large) and just two sizes of third (4 and 5 semitones large). It is enormously important, and was once—prior to the spread of Western music—the world’s most popular scale.

C pentatonic

The *whole-tone scale* divides the octave into six precisely even parts and contains as many major thirds as a three-note scale can have. (That is, every note in the scale has a major third above it.) It was first used by Glinka, and was made famous by Debussy. It frequently appears in jazz (especially Thelonious Monk’s playing) and is often used to signify dream sequences in television and movies.



If you transpose the scale by two chromatic semitones, you end up with the same scale again—meaning that there are just two whole-tone scales! Since this scale has very little internal variety, it quickly becomes boring.

A second six-note scale is the *hexatonic scale*; consisting of alternating one-semitone and three-semitone steps. It has as many major thirds as a scale can have (one above each note!) while also having three different perfect fifths and containing many different major and minor triads (how many, if you ignore spelling?)



Among the important seven-note scales, we have the *ascending melodic minor* or *acoustic scale* (also sometimes called the “lydian dominant”), with major or minor thirds above each of its notes, and which divides the octave fairly evenly. It gets this name because it is the best approximation to the first 7 pitch classes in the overtone series.

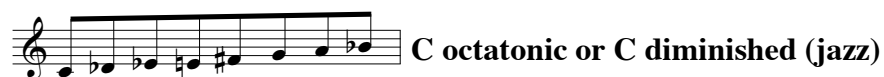


The ascending melodic minor has steps of 1 and 2 semitones and thirds of 3 and 4 semitones, just like the diatonic scale. The fourths, however, come in *three* sizes: 4 semitones (B-E $\flat$), 5 semitones (D-G), and 6 semitones (E $\flat$ -A).

There are two other important seven-note scales, the harmonic minor and harmonic major scales. These scales have *three* step sizes: each scale has a single three-semitone step, as shown on the bracket. However, the scales are maximally saturated with major and minor thirds, just like the diatonic scale.



Finally, there is the *octatonic scale*, which is maximally saturated with minor thirds, while also containing many perfect fifths and triads (how many?).



The octatonic scale has two sizes of step (1 and 2 semitones) and fourth (4 and 5 semitones). However, it has only one size of third (3 semitones) and fifth (6 semitones). In this sense it is nearly as symmetrical as the whole-tone scale: if we transpose the octatonic scale by three semitones, then we end up with the same scale again.

The following table summarizes our scales.

# of notes	Very even	Somewhat Less Even	Maximally Saturated with
Five notes	Pentatonic		Perfect fifths
Six notes	Whole tone	Hexatonic	Major thirds
Seven notes	Diatonic	Melodic minor ascending, Harmonic minor Harmonic major	Perfect fifths, Thirds Thirds Thirds
Eight notes	Octatonic		Minor thirds

VI. A brief history of scales in Western music.

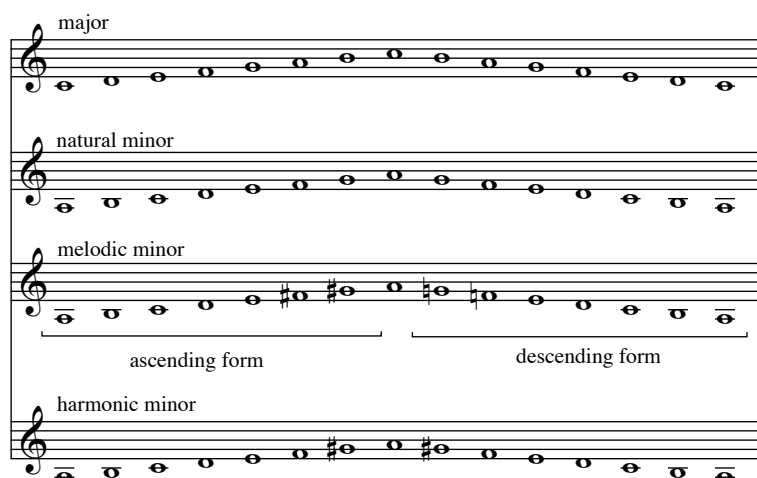
a) Medieval and Renaissance music (1100–1600) uses only the diatonic scale.

Cadences on various scale degrees suggest all the modes except Locrian. Accidentals are rare, so the music typically employs the “white note” diatonic collection (C major/A natural minor). Unsystematic use of sharps and flats occasionally gives rise to other diatonic scales however.¹

b) Classical tonality (1600–1875) limits the number of modes to two: Ionian or major, and aeolian or “natural minor.” The desire for a “leading tone” (the note a semitone below the tonic) leads to three separate forms of the minor scale. Minor-key music switches freely (and rapidly) between these three forms.

Scale degrees of major and minor are often named according to the following system: the first is the *tonic*, the second is the *supertonic*, the third is the *mediant*, the fourth is the *subdominant*, the fifth is the *dominant*, the sixth is the *submediant*, and the seventh is the *leading tone* (when it is one semitone below the tonic) or the *subtonic* (when it is two semitones below the tonic). **You have to learn these names.**

¹ This is a bit of an approximation. We know that performers often added extra accidentals when performing this music; but we don’t know how often they did so. It’s possible that the music we think is purely diatonic was actually performed in a way that we would consider very chromatic!



c) The modern period (1875-now). In the late nineteenth century, composers became newly interested in the diatonic modes; this reawakening continued in the twentieth century. Since there were now 12 major scales, and each had 7 modes, we now had $12 \times 7 = 84$ different modes!

Composers also became interested in all the nondiatonic scales shown on this handout, as well as all of their modes. (Note that highly symmetrical scales such as the whole tone and octatonic do not give rise to many distinct mode-types—all of the modes of the whole-tone scale belong to the same type; similarly, the octatonic and hexatonic scales have only two distinct mode types apiece.) Musicians such as Debussy and Ravel, modern jazz improvisers, and minimalists such as Steve Reich, have developed an impressive facility with all the modes of all the different scales discussed here. We will explore this further in the second semester.

VII. Tutorial on identifying scales. You should get to know, and be prepared to identify, all of the scales on this handout. Here's how to do it.

1. Put all the pitch classes of the scale in order, beginning and ending with the same note. For instance: $C\sharp-D-E-F\sharp-G-A-B\flat-[C\sharp]$.

2. Identify the sequence of chromatic intervals between the successive notes. For instance: 1-2-2-1-2-1-3.

3. Rotate this sequence of intervals until it matches one on the following table, which you simply have to memorize. For instance, we can rotate move the first element of 1-2-2-1-2-1-3 to the end, producing 2-2-1-2-1-3, which matches the “harmonic major” scale.

4. The first note of the rotated ordering gives you the name of the scale. For example, we call the circular ordering of notes $D-E-F\sharp-G-A-B\flat-C\sharp-[D]$ the “D harmonic major scale.” (Another way of saying this is that we name the scales relative to the ordering shown on this handout. This is just a convention; it doesn't mean that these orderings are better than the others!)

Number of Notes	Scale	Sequence of Intervals
Five	Pentatonic	2-2-3-2-3
Six	Whole tone	2-2-2-2-2-2
	Hexatonic	1-3-1-3-1-3
Seven	Diatonic	2-2-1-2-2-2-1
	Melodic Minor Ascending	2-1-2-2-2-2-1
	Harmonic minor	2-1-2-2-1-3-1
	Harmonic major	2-2-1-2-1-3-1
Eight	Octatonic	2-1-2-1-2-1-2-1

5. It's relatively easy to identify the five, six, and eight-note scales. Here are some hints that may help untangle the seven-notes scales.

a) The diatonic scale has five 2s and two 1s. The two 1s are as far away from each other as they can be—they have at least two 2s between them. The name of the diatonic scale is given by the lowest note that forms the isolated pair of 2s.

b) The ascending melodic minor ascending scale also has five 2s and two 1s. The 1s are not as far away from each other as they can be—they have only one 2 between them. The name of the melodic minor ascending scale is given by the lowest note of the isolated 2.

c) The harmonic major and minor scales have only one 3-semitone interval. Find the note that is the top of the three-semitone interval, and go one scale step above it. This note determines the name of the scale. So, in D-E-F $\sharp$ -G-A-B $\flat$ -C $\sharp$ -D, the 3-semitone interval is B $\flat$ -C $\sharp$. The note one step above the top note is D, so we have a D harmonic scale of some kind. To figure out whether the scale is major or minor, look at the third above the D—if it's major, the scale is harmonic major; if it's minor, the scale is harmonic minor. So: the note a third above D is F $\sharp$; D-F $\sharp$ is a major third, so our scale is a D harmonic major scale.

THE DIATONIC MODES

There are seven modes of the diatonic scale. They all contain the same notes, but emphasize a different note in the collection. The main note of each mode is the *tonic* of the mode.

in this column, each mode is transposed so that F is tonic

4. F lydian	F is tonic	F lydian
1. C ionian (= major)	C is tonic	F ionian
5. G mixolydian	G is tonic	F mixolydian
2. D dorian	D is tonic	F dorian
6. A aolian (A natural minor)	A is tonic	F aolian
3. E phrygian	E is tonic	F phrygian
7. B locrian (very rare)	B is tonic	F locrian

THE MODES OF THE MELODIC MINOR

There are seven modes of the ascending melodic minor scale. (Here I've listed them in two ways: first, as different arrangements of the notes C-D-E $\flat$ -F-G-A-B, and second, so that they start on C.) Some of them are very common in jazz and other musics, where they are used in both their ascending and descending forms. Note that these modes are often called "scales."

1. C melodic minor ascending



2. not named (rare)



3. not named (rare)



4. F Lydian Dominant scale (jazz), F Acoustic Scale, F Lydian/Mixolydian scale



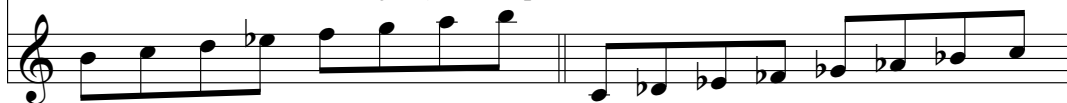
5. not named, but fairly common



6. The A locrian $\sharp 2$ scale (sharp-two, not number two)



7. The B altered scale (jazz); B super locrian



DIATONIC KEY SIGANTURES

C major	G major	D major	A major	E major	B major	F $\sharp$ major	C $\sharp$ major
A minor	E minor	B minor	F $\sharp$ minor	C $\sharp$ minor	G $\sharp$ minor	D $\sharp$ minor	A $\sharp$ minor

C $\flat$ major	G $\flat$ major	D $\flat$ major	A $\flat$ major	E $\flat$ major	B $\flat$ major	F major	C major
A $\flat$ minor	E $\flat$ minor	B $\flat$ minor	F minor	C minor	G minor	D minor	A minor

Note: there are no key signatures requiring a double sharp or double flat. This is why we don't find, for instance, F $\flat$ major, whose notes F $\flat$, G $\flat$, A $\flat$, B $\flat\flat$, C $\flat$, D $\flat$, E $\flat$ requires a double flat.

ROMAN NUMERAL ANALYSIS

I. Three-note consonances. Early music theorists divided the intervals into *consonances* (unison, minor third, major third, perfect fourth, and all their registral inversions¹) and *dissonances* (everything else). Let's take a look at the three-note chords containing only consonances.



Those in (a) have two copies of some note. As a result, they sound a little less rich and full than those in (b). We call this second group *triads*, since they have three distinct pitch classes. Over the centuries musicians gravitated more and more toward triads, preferring their richness to the empty sound of the chords in (a).

II. Figured bass symbols. Originally, triads were named by giving their bass note, plus their intervals above the bass. The symbol $\overset{5}{3}$ means that there are a fifth and third above the bass, $\overset{6}{3}$ means that there are a sixth and third above the bass, $\overset{6}{4}$ means that there are a sixth and fourth above the bass. (Here we again disregard extra octaves: “a fifth” could mean “fifth plus an octave” or “fifth plus two octaves,” etc.) So in the music below, the first three chords would have been called $C\overset{5}{3}$, the second $A\overset{6}{3}$, and the third $B\overset{6}{4}$.

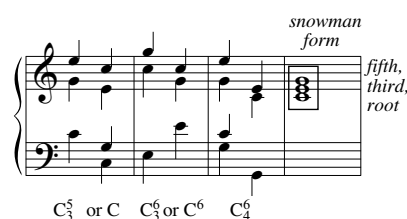


It is customary to abbreviate these symbols by eliminating the $\overset{5}{3}$, writing $\overset{6}{3}$ for $\overset{6}{3}$, and keeping $\overset{6}{4}$ unchanged.

¹ That is, all the notes you could get by starting with U, m3, M3, P4 and transposing either note by any number of octaves.

III. Roots. When you classify chords in this way, there appear to be three fundamental kinds of consonant triads: $\frac{5}{3}$, $\frac{6}{3}$ and $\frac{6}{4}$. In the early eighteenth century, Jean-Phillippe Rameau pointed out that these could all be understood as different forms of a single chord. His central idea was to classify chords on the basis of the *notes they contain* rather than intervals above the bass. Thus, if we have {C, E, G} with C in the bass, then this is a $C\frac{5}{3}$ chord. If E is in the bass instead, then this is a $C\frac{6}{3}$ chord. And if G is in the bass, then this is a $C\frac{6}{4}$ chord. So the “three kinds of consonant chord” can be reduced to different arrangements of the notes of one chord. This was a great intellectual triumph.

Following Rameau, we now say that the notes {C, E, G} are a “C major chord” no matter what note is in the bass. We say that C is the *root* of the chord, E is the *third*, and G is the *fifth*. **We now label chords based on their roots rather than their bass note;** however, we continue to use the numerical symbols in Section II, as follows:



To find the root, fifth, and third of a chord, put it onto *snowman form*: that is, transpose notes by octave until they lie on three adjacent staff lines, or three adjacent staff spaces. When the chord looks like a snowman, the lowest note is the root, the middle note is the third, and the highest note is the fifth. The root determines the name of the chord.

IV. Triads. In snowman form, major and minor triads are both stacks of thirds: a major triad has a major third on the bottom and a minor third on top; a minor triad has a minor third on the bottom and a major third on top. (A minor triad is a kind of “upside down” major triad.) There are two other ways to stack major and minor thirds. Two major thirds (such as C-E and E-G $\sharp$) form an *augmented triad*, and two minor thirds (such as C-E $\flat$ and E $\flat$ -G $\flat$) form a *diminished triad*. When in snowman form, an augmented triad has an augmented fifth; the major and minor triads have a perfect fifth; and the diminished triad has a diminished fifth.

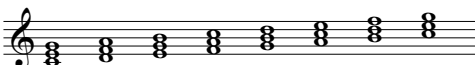
	“stack of thirds”	Third	Fifth	Example	symbol
Augmented	M3 + M3	M3	A5	{C, E, G $\sharp$ }	C+
Major	M3 + m3	M3	P5	{C, E, G}	C
Minor	m3 + M3	m3	P5	{C, E $\flat$, G}	c
Diminished	m3 + m3	m3	d5	{C, E $\flat$, G $\flat$ }	c $^\circ$

In general augmented triads are labeled with a capital letter (corresponding to the root of the triad) and plus sign; major triads with a capital letter; minor triads with a small letter; and diminished triads with a small letter and a degree sign.

Note that the diminished triad has a dissonant interval, the diminished fifth. The augmented triad's augmented fifth was considered dissonant as well, even though it sounds just like a minor sixth. (This might be confusing; however, in specific musical contexts, an augmented fifth usually sounds unstable, and like it wants to resolve to some other interval.) Though dissonant, augmented and diminished triads gradually gained acceptance because of their similarity to the consonant major and minor triads.

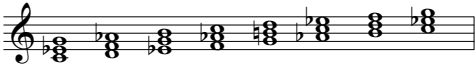
V. Roman numerals. When we know what key we're in, we notate chords using Roman numerals indicating the scale degree of the triad's root. A triad on the first scale degree is labeled I (for major) or i (for minor), a triad on the second scale degree is labeled II (for major) or ii (for minor), and so on. Once again, a capital letter with a plus sign (e.g. III+) indicates an augmented triad, a capital letter (e.g. III) indicates a major triad, a small letter (iii) indicates minor, and a small letter with a degree sign (iii°) indicates diminished. We indicate the key to the left of the Roman numerals, using capital letters for major keys and small letters for minor keys.

major



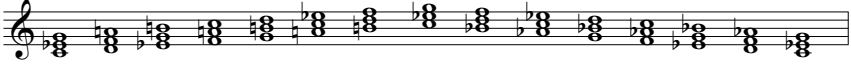
C: I ii iii IV V vi vii° I

minor (harmonic minor scale)



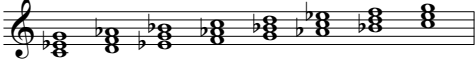
c: i ii° III+ iv V VI vii° i

minor (melodic minor scale)



c: i ii III+ IV V vi° vii° i VII VI v iv III ii° i

minor (natural minor scale)



c: i ii° III iv v VI VII i

We can use the same system for modal music, though you should specify the mode completely:

mixolydian



C mixolydian: I ii iii° IV v vi VII I

Roman numerals can also be used for chords not in the scale: accidentals before the numeral apply to the root, with capital/small letters referring to major/minor in the standard way. Thus “III” refers to an E major chord in the key of C major (iii with a raised third), and “bVI” refers to F major in the key of A major. Similarly, “#ii°” in G major is an A# diminished triad.

VI. Seventh chords. Though we will not be using seventh chords for a few more weeks, it's worth putting them here.

	"stack of thirds"	Third	Fifth	Seventh	Example	symbol
Major seventh	M3 + m3 + M3	M3	P5	M7	C, E, G, B	C ^{maj7}
Dominant seventh	M3 + m3 + m3	M3	P5	m7	C, E, G, B $\flat$	C ⁷
Minor seventh	m3 + M3 + m3	M3	P5	m7	C, E $\flat$, G, B $\flat$	c ⁷
Half-diminished seventh	m3 + m3 + M3	m3	d5	m7	C, E $\flat$, G $\flat$, B $\flat$	c ^{ø7}
Diminished seventh	m3 + m3 + m3	m3	d5	d7	C, E $\flat$, G $\flat$, A	c ^{o7}
<i>Augmented/major seventh</i>	M3 + M3 + m3	M3	A5	M7	C, E, G $\sharp$, B	C ⁺ maj7
<i>Minor/major seventh</i>	m3 + M3+M3	m3	P5	M7	C, E $\flat$, G	c ^{maj7}

(Chords in italics are rare in classical music.) Here are the seventh chords in the familiar major and minor scales:

major

C: I^{maj7} ii⁷ iii⁷ IV^{maj7} V⁷ vi⁷ vii^{ø7} I^{maj7}

minor (harmonic minor scale)

c: i^{maj7} ii^{ø7} III⁺maj7 iv⁷ V⁷ VI^{maj7} vii^{ø7} i^{maj7}

minor (melodic minor scale)

c: i^{maj7} ii⁷ III⁺maj7 IV⁷ V⁷ vi^{ø7} vii^{ø7} i^{maj7} VII⁷ VI^{maj7} v⁷ iv⁷ III⁺maj7 ii^{ø7} i⁷

minor (natural minor scale)

c: i⁷ ii^{ø7} III⁺maj7 iv⁷ v⁷ VI^{maj7} VII⁷ i⁷

VII. Roman numeral analysis. We will often ask you to engage in “Roman numeral analysis.” To do this, you need to write Roman numerals *and figured bass symbols* under the chords in a piece of music, making sure you indicate the music’s key or mode.

C: I IV⁶ V⁶ I I⁶ IV V I⁶

Fluency with Roman numeral analysis is one of the primary skills you will learn in this course. You need to be able to quickly recognize the Roman numerals when looking at a score; you also need to be able to *hear* them when listening to a piece.

¹ Passing tones, neighboring tones, suspensions, anticipations, pedal tones, and incomplete neighbors.

(“intro”) or conclusion section (“outro”). For complicated songs, you can always use the letters A, B, C, etc. to indicate the form (“ABACAB”).

III. Three-chord songs: major

In baroque and classical music, chords typically progress according to very rigid rules. For instance, the IV chord often goes to V, whereas the V chord almost never goes to (root position) IV. These conventions are known as the principles of *functional harmony*, and we’ll be studying them in just a few weeks.

Contemporary popular music is not like this: in this style, virtually any chord can move to any other. The same can be said for a range of other genres, including Renaissance music and contemporary classical music. In fact, “any chord can follow any other” is something of the default for musical styles; the rigid harmonic rules of the classical period are unusual.

Popular songs tend to be fairly simple harmonically. Many, for example, revolve around the three major chords in the diatonic scale: e.g. the C, F, and G major triads in C diatonic. In principle, you can make any of these chords the tonic, and you can put them together in any order:

	Tonic	Mode	Roman Numerals	Example
C-F-G-C	C	Ionian	I-IV-V-I	“Twist and Shout” (Various/Beatles) “Blitzkrieg Bop” (Ramones)
C-G-F-C	C	Ionian	I-V-IV-I	“Baba O’Riley” (The Who) “Summer Babe” (Pavement)
G-F-C-G	G	Mixolydian	I-VII-IV-I	“I Can’t Explain” (The Who) “Sympathy for the Devil” (Rolling Stones)
G-C-F-C	G	Mixolydian	I-VII-IV-I	“Tears of a Clown” (Various)
F-G-C-F	F	Lydian	I-II-V-I	??
F-C-G-F	F	Lydian	I-V-II-I	??

It is interesting that the Lydian options are extremely rare: on examination, almost all of the purportedly “Lydian” songs turn out to be in some other mode. If any of you can find a genuinely Lydian song, I’d be very happy.

Note that songs can switch modes. For instance, “I Can’t Explain” has a verse in E mixolydian: E-D-A-E (I-VII-IV-I). It then switches to E ionian for the chorus: E-c[#]-A-B (I-vi-IV-V). Here the *mode* switches from mixolydian and ionian, while the *tonic* stays the same (E in both cases). The same thing happens in hundreds of other songs, including “Sympathy for the Devil,” “Turn the Page,” etc.

IV. Minor key harmony

Classical music is unusual in that it reuses virtually the same chord progressions for the two modes: if a progression is acceptable in major (for instance, I-IV-V-I) then it typically has an

analogue in minor (i-iv-V-i). In popular music, minor pieces use their own unique repertoire of chord progressions.²

Many minor songs involve the contrast between the tonic and the major chord two semitones below: i-VII-i (“Drunken Sailor,” “Scarborough Fair,” “Sounds of Silence,” etc.). A number of songs add the VI chord, and are arranged around some variant of a i-VII-VI-VII-i pattern (“All Along The Watchtower,” “Gimme Shelter”). Still others involve the III chord, as in i-III-VI-VII-i (“Mongoloid,” “Get Into the Groove”). All of these chords are in the natural minor scale or aeolian mode.

Alternatively, there are songs that combine i, VII, III, and IV, producing a dorian-mode feel (“Born to be Wild,” “Turn the Page” [Seeger], “Mad World,” etc.) Note that i, VII, and III are common to both aeolian and dorian; the difference is the VI chord (which is unique to natural minor) and the IV (which is unique to dorian).

It is interesting that these chord progressions combine a minor chord with one or more of the three major chords in the scale: a-G-a, a-G-F-a, a-C-F-G-a, d-F-C-G-d, etc. It is almost as if the major-mode emphasis on C, F, and G has been incorporated into a minor-key context, with A minor or D minor as tonic.

V. Rock rhythm

The basic unit of rock rhythm is a four-unit pattern, with the hi-hat³ playing on all four beats, the bass drum on beat one, and the snare drum on beat three.

Hi hat	X	X	X	X
Snare			X	
Bass	X			

Since the music is not notated, this leaves us with a puzzle: is the music in a fast 4/4, with the snare on the third quarter note, or is it in a slower 2/4, with the snare on the second beat? (Alternatively, is it in a slow 4/4, with the above pattern representing just half a measure?) In other words, is the main pulse given by the hi hat or the combination of bass and snare?

There are several ways we can study this: we can examine how musicians count when introducing a song; we can consider the styles that gave birth to rock, which use very similar patterns; we can examine the length of individual sections⁴; and we can compare the tempo distributions of rock songs to those of other styles.

Historically, the answer is that the above pattern is only half a measure, and that the snare is on beat 2 and 4 of a 4/4 measure. (This is how educated listeners typically count.) Rock tempi, understood in this way, are significantly slower than those in other styles!

² One can sometimes find minor-mode songs (e.g. “Paint it Black”) that use analogues of major-mode progressions, as in classical music.

³ A kind of cymbal, which can be opened and closed with a foot pedal.

⁴ This would tell us whether the basic pattern always appears in multiples of two, which would indicate that it represents only half of a full measure.

MUSIC 105

Dmitri Tymoczko
Handout 9 (2010)

OUCH! OR, WRITING FOUR-PART COUNTERPOINT

(while also avoiding forbidden parallels!)

In four-voice triadic writing, the upper three voices are typically found in one of three positions: *close position*, where they sound a complete triad and are as close together as possible (e.g. C4-E4-G4), *half-open position*, where tenor and soprano are an octave apart with alto sounding a chord tone between them (e.g. G3-E4-G4), and *open position* where there is one unsounded chord tone between each of the three upper voices (e.g. G3-E4-C5, where chord tones C4 and G4 are not sounded). Note that the terms “close position” and “open position” are common; “half-open” is my own. (As far as I know there’s no accepted term for this arrangement.) Together these three positions account for more than 95% of the upper-voice configurations in a typical Bach chorale. If we add a fourth category (“unusual,” referring to anything else), we have four possibilities, which you can remember with the mnemonic OUCH [open, unusual, close, half-open].



Here is the fundamental fact you need to know: *one half-open voicing cannot progress to another; this creates parallel octaves. Half-open voicings therefore typically move either to close position or to open position. By contrast, you can use the “3 + 1” trick (see below) to move from one close-position voicing to another, or from one open-position voicing to another.*

I. The “3 + 1” Trick

This is the fastest and safest way to write four-part harmony while also avoiding parallels. The basic idea is to arrange the upper three voices so that they sound a complete triad in close position. The bass sounds either the root or the third, though root is most typical. The only rule is that if the upper three voices, *considered on their own*, are sounding a root position triad, then they cannot move to another root position triad. (If they do, it creates parallel fifths.) As long as you obey this rule, the upper voices will never produce forbidden parallels.

This means you only need to check for parallels between the upper voices and the bass. The easiest way to avoid these parallels is to move the bass in contrary motion to the upper voices. By following the contrary motion rule, you’re guaranteed to avoid forbidden parallels as long as the voice leading doesn’t have crazy leaps. However, there are many situations where it’s OK to move the bass and upper voices in the same direction: for instance, when the chords are moving by perfect fourth (e.g. C major to F major) and the bass is sounding chord roots.

This schema is *very useful* for exams; in fact, you're pretty much doomed if you don't manage to learn it. However, with a little practice, you should be able to use this technique to harmonize almost instantaneously.



There are two ways to embellish the schema.

First, you can transpose the middle of the upper three voices down by octave, so that the chords are in open position—that is, there is one unsounded chord tone between each voice. (For instance, when tenor, alto, soprano sound G3, D4, B4, then there is an unsounded B3 between the first two notes and an unsounded G4 between the second two.) Note that this makes it slightly easier to create parallels: now, you cannot move the upper three voices move from one first-inversion triad to another. This means there are two disallowed moves: root position to root position and first inversion to first inversion. (Remember, I'm talking here about the upper voices *considered in isolation from the bass*.)



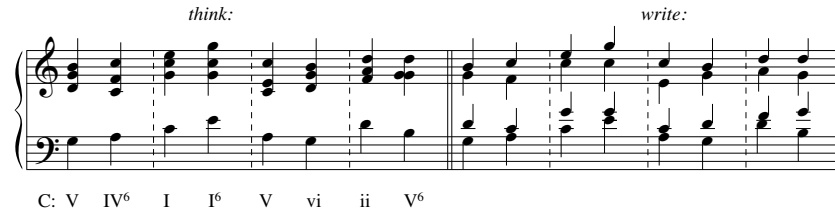
The second embellishment is to make the “3+1” division so that the bass voice participates in the 3-voice scheme. For instance, in the first voice leading in the third measure, above, the tenor leaps between chord roots, while the rest of the voices articulate the three-voice voice leading between complete triads. (Compare this to the first voice leading in the first measure: I've just switched bass and tenor, and changed octaves.)

About 60% of the voice leadings in the Bach chorales are of this form. In some music, the percentage is much higher.

II. Using half-open chords.

In half-open position, the upper voices, on their own, sound an incomplete triad; the bass voice adds the missing note (e.g. upper voices have C4-F4-C5 while bass adds A3, completing the F major triad). One half-open voicing never moves to another half-open voicing; hence, you must move from either open or close to half-open, and from half-open to either open or close.

Half-open to close (or vice versa) will never create parallels; however, half-open to open (or vice versa) occasionally can; to check for this, look at what happens to the perfect fifth (if any) in the half-open chord.



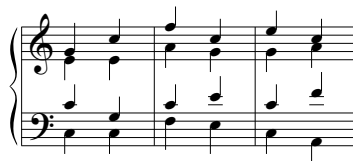
NOTE: Each measure is its own example.

About 15% of the voice leadings in the Bach chorales are of this form.

As the above example indicates, you will occasionally find the upper voices in some other position—for instance, with two voices sounding the same pitch (G4-G4-D4) or with the octave between tenor and alto (D3-D4-G4). These positions act like the “half-open” position in the following sense: they typically move to an open or close position, in which the upper voices are nice and evenly spaced and sounding a complete triad. *Sometimes*, however, you may find one of these “unusual” voicings moving to a half-open position or vice versa.

III. Moving between close and open positions

Periodically, a close-position voicing (in the upper voices) moves to an open position voicing or vice versa. Here there won’t be parallels in the upper voices; you only need to check for parallels with the bass.



IV. Why use these tricks?

If you don’t use the tricks, you need to check for parallels between *every pair of voices*. There are six of these pairs, so this is a lot of checking. However, if you use the tricks, then you typically only need to check for parallels between upper voices and bass. This makes it significantly easier to avoid parallels.

V. Adding voice crossings.

You can also embellish either of the first two schemas by adding voice crossings. This can create extra melodic interest, while also giving you a quick-and-dirty way to avoid parallels.

Examine the opening of Palestrina’s “Adoramus, Te,” illustrated below. The lowest staff illustrates the basic voice-leading patterns. The first five voice leadings are all based on the “3+1” trick—embellished, in two cases, with a crossing. (Without that crossing, note that would be parallel fifths and octaves!) The last two use a variant of the half-open technique, since the upper voices move from a complete triad (G-C-E) to an incomplete triad (A-A-D) to a complete triad (G $\sharp$ -B-E). (Note that here, the alto and tenor sound the same note; in a true half-open

voicing, the tenor and soprano are an octave apart, sounding the same *pitch class*.) Palestrina uses a number of “nonharmonic tones” in the fourth bar; we’ll discuss this in the next few classes. For now, you can just think of the underlying harmony here as being F-A-A-D.

the 3+1 trick
variant of the
crossing
half-open

voicing

V. Do what you want!

Don’t feel constricted by these techniques—feel free to let your imagination roam. But remember, that they together account for a fairly large proportion (95%) of voice leadings in a wide range of music. You should think of them as “defaults” that allow you to compose quickly when you have to, or when you don’t have a better idea.

STYLE 2: FOUR-VOICE RENAISSANCE COUNTERPOINT

Our second musical style is modeled after the polyphonic music of the Renaissance, in particular the music of the period 1450-1600. The basic harmonic objects are triads, though they are sometimes stated incompletely. Except at cadences, chords can progress freely—unlike later tonal music, there are no rules governing harmonic progressions *per se*. However, there are a variety of new contrapuntal and harmonic restrictions. Two of these are fundamental: the rejection of parallel perfect consonances, and the reclassification of the fourth as a (tolerable) dissonance.

I. Pitch. The music should use the triads belonging to a single diatonic scale. In certain circumstances (see VI, below) you can add accidentals at a cadence.

II. Rhythm. All voices sing in the same rhythm. In general the chord will change on each quarter note. The music should be written in 2/4, 3/4, or 4/4. Cadences should occur on the strong beats of the measure—that is, beat 1 in 2/4 and 3/4, and beats 1 or 3 in 4/4.

III. Voices and ranges. *Unless otherwise noted, the music should be written for four voices: soprano, alto, tenor, and bass.* Observe the standard vocal ranges. (Soprano: C4 ↔ G5, Alto: G3 ↔ C5, Tenor: C3 ↔ G4, Bass: G2 ↔ C4.) In 3- or 4-voice textures, the upper voices should never be more than an octave apart. The bass voice can sometimes be a twelfth (octave plus fifth) below the next highest voice. These rules can be violated briefly and very occasionally, with compelling artistic justification.

IV. Counterpoint. Each voice should make its own coherent melody. Pay particular attention to the outer voices (e.g. soprano and bass). Inner voices are sometimes allowed to be a bit boring. In general, voices should move by step and avoid large or awkward leaps; common tones should be held in the same voice. The bass voice will tend to leap more than the upper voices. **In general the bass voice should not repeat notes.**

A. Rules governing individual lines.

1. *Forbidden intervals.* No voice should ever move by a seventh, more than an octave, or by any augmented interval.

2. *Intervals requiring a change of direction.* Diminished intervals may only be used if the melody changes direction after the interval. A melodic interval larger than a P4 should typically be left in the direction opposite to the leap.

3. *Consecutive leaps.* Consecutive leaps in the same direction should outline a triad; thus, consecutive fourths in the same direction are forbidden.

B. Rules governing forbidden parallels.

1) *Consecutive perfect intervals*: no two voices can make perfect octaves on two consecutive chords, or perfect fifths on two consecutive chords. Example: it is forbidden for the bass to move from G3 to C3, while alto moves from D3 to G4.

2) *Diminished fifth to perfect fifth*. If some voice is a diminished fifth above the bass, then it cannot make a perfect fifth with the bass on the next chord. [NB: This rule will only be relevant later, when we allow seventh chords; I'm listing it here so that all of your counterpoint rules are in one place.]

3) *Direct fifths*. When the top voice leaps into a note that forms a perfect consonance with the bass, the bass must move by contrary motion into that same interval. Example: it is forbidden for the soprano to make an ascending leap from A4 to D5 while bass ascends from F3 to G3.

C. Crossing and overlap

1) *Voice crossing*. No voice should ever cross above the soprano or below the bass. In cases of artistic necessity, the inner voices can briefly cross.

2) *Voice overlap*. Voices should not cross the previous position of a neighboring voice; that is, it should not move in a way that would create a crossing if the neighboring voice stayed fixed. Example: if the tenor has C4 and the alto has E4, then the alto should not move below C4 on the next chord, no matter where the tenor actually moves. (This rule can occasionally be broken.)

V. Harmony.

A. Triads.

1. *Complete triads*. In four voices, almost every harmony should contain a complete triad; with fewer voices you will have to use more incomplete triads.

2. *Incomplete triads*. In three or more voices, incomplete triads should consist of root and third. Bare perfect consonances should occur only at a cadence.

B. Consonance and dissonance.

1. *Consonance and dissonance*. See the following table. Compound intervals are classified in the same way as non-compound intervals: tenths are imperfect consonances, twelfths are perfect, etc.

perfect unison, perfect fifth	perfect consonance
major or minor third or sixth	imperfect consonance
perfect fourth	<i>dissonant</i> if the lowest note of the interval is the bass voice; consonant otherwise
augmented fourth, diminished fifth	<i>tolerable dissonance</i> if the bass is not involved; intolerable otherwise
everything else	dissonance

2. As a result, triads can appear only in root position or first inversion. *Second inversion triads are considered dissonant!*

IV. Inversions. Phrases should begin and end with root position triads. Generally, root position chords are the norm—use them about 2/3 of the time. First inversion triads should be used only when they contribute to a smooth, melodic bass line, about 1/3 of the time. A good rule of thumb is: when the third of the chord is in the bass, it should be connected by step to either the note that comes before it, or the note that comes afterward. (Sometimes one of these notes will be a passing tone.) However, this is just a rule of thumb—the arpeggiation in the third measure below is perfectly normal.

bad: leaping between inversions without a clear melodic or harmonic purpose.

good: the stepwise bass is melodic

good: the bass arpeggiates an A minor chord, which is also the chord in the upper voices

V. Cadences. In general phrases, should begin and end with the same chord, though this rule can occasionally be violated.

1. *Three or more voices:* At every cadence, two voices should move by step, in contrary motion, to a unison or octave. The pitch class on which these voices converge should also appear in the bass of the final chord. The remaining voices should be deployed so that the penultimate chord is a complete triad. In four voices, the final chord can be either a root position triad, a bare third, or a bare fifth. In three voices, it can also be a unison. Example: tenor moves from B3 to C4, while alto moves from D4 to C4; bass moves from G3 to C3. Or: alto moves from D4 to E4 while bass moves from F3 to E3 and tenor moves from A3 to B3.
2. *Two voices:* you can use any two voices of an acceptable 3-voice pattern, so long as the two voices you use do not violate any other rules. Example: tenor moves from B3 to C4, while bass moves from G3 to C3. Or: tenor moves A3 to B3 while bass moves F3 to E3.
3. *Leading tones.* If, at a cadence, two voices move in contrary motion by a *major second* to a unison or octave, add an accidental so that the ascending voice ascends by *semitone*. Be sure that in doing so, however, you avoid augmented seconds in the melody! Also, do not double a raised tone! Example: at the cadence, bass moves F#3→E3 while alto moves D4→E4; one should then change the alto to D#4→E4. This D#4 should not appear in any other voice.

NB: to avoid augmented seconds, it may sometimes be necessary to add an accidental two chords before the final chord. Example: at the cadence, bass moves A3→F#3→E3 while alto moved C4→D4→E4; one should then change the alto to C#4→D#4→E4.

Embellishing Chorale-style Harmony with Nonharmonic Tones

There are a number of ways to take a simple voice-leading skeleton and embellish it to create something more sophisticated and musical. Most of these involve *nonharmonic tones*, or notes foreign to the underlying harmony. There are four main types of nonharmonic tones in Renaissance music and classical chorale-style harmony, the *passing tone*, *neighboring tone*, *anticipation*, and *suspension* (“PNAS”).¹

I. Chordal skips. Before discussing nonharmonic tones proper, let me mention the *chordal skip*, in which a voice leaps from one note to another, usually on a weak eighth note. This can be used to add rhythmic variety or to move from one position to another.



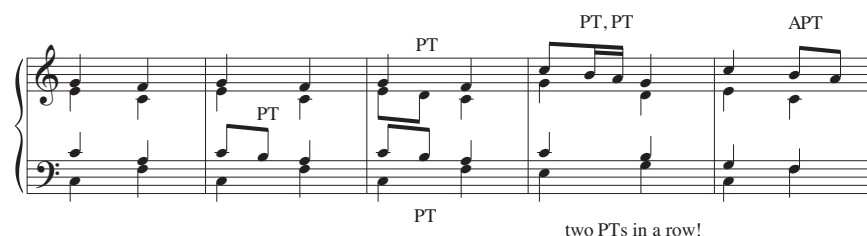
Although it is not generally a good idea, composers sometimes use the chordal skip to evade parallel fifths and octaves. Cf. tenor and bass, beats 3-4 in measure 1 of the following:



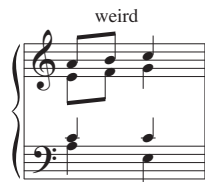
If there were no chordal skip on the weak eighth, we would have parallel fifths; and in fact, the (cultivated) ear will still hear a trace of “fifthness” in this progression, because of the fifths on consecutive strong beats. Don’t rely on this trick; it is a very rare occurrence in actual music, and your preceptor may start to penalize you if you overuse it! However, in a pinch, it’s better than nothing ...

II. Passing tones. A passing tone moves from a chord tone to another chord tone by step and in the same direction, producing a dissonance along the way. Most passing tones occur on a weak beat (in chorale style, almost always the second eighth note in a beat), and usually connect two consonances. However, you sometimes find two passing tones in a row, as in the fourth example below. Sometimes the passing tone occurs on a strong beat, forming an *accented passing tone*.

¹ There are actually a few others that you will eventually need to learn: incomplete neighbor tones, pedal tones, and—if you dig deep into Renaissance counterpoint—the “cambiata.”



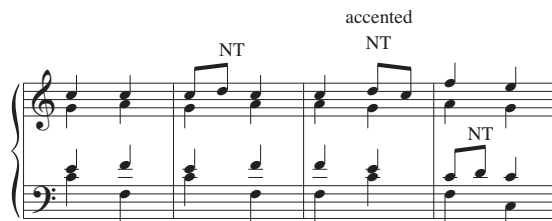
If multiple voices create passing motion, it is a good idea for the passing tones to be consonant with each other. Bare or obvious parallel fourths, caused by passing tones in two voices, are fairly rare:



PTs don't sound very consonant together.

Sometimes a composer will deliberately use passing tones to create a dissonant “clash,” as in the F minor fugue from Book I of the *Well Tempered Clavier*. This is a special effect, though, and is relatively unusual. Not recommended until you get to MUS206 or so.

III. Neighbor tones. A neighbor tone moves by step from a consonance to a dissonance, before reversing direction and returning by step to the *original* consonance. Neighbor tones can be accented or unaccented.

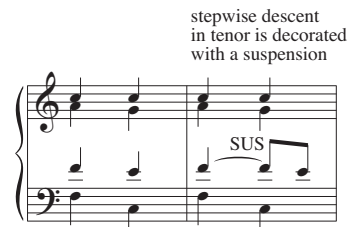


IV. Anticipations. Anticipations arise from rhythmic displacement: a chord tone appears a little bit before the chord. Anticipations most often occur in the melody, and usually move by step. However, one does find them in other voices and—very occasionally—they may involve leaps.

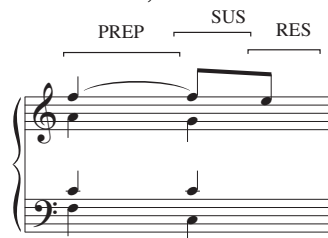


Anticipations are almost always unaccented, since they precede the arrival of a chord.

V. Suspensions. Suspensions are the most complicated of the nonharmonic tones. Intuitively, you can think of them as a way to decorate a descending melodic step (in any voice) by delaying the arrival of the second note.



Formally, the suspension has three phases, the *preparation*, *suspension*, and *resolution*. In the preparation, the note is a consonance. In the suspension phase, that note is elongated, or held over to become a dissonance; in the resolution, the note moves *down by step* to a consonance.



The two basic principles are (1) the preparation must be *at least as long* as the suspension² (though it can be longer); and (2) the suspension must be *rhythmically stronger than the resolution*. Thus, in a standard 4/4 measure, a half-note suspension should occur on beat 1, resolving on beat 3; a quarter-note suspension can occur on beats 1 or 3, resolving on beats 2 or 4; an eighth-note suspension can occur on the first eighth note of *any* beat, resolving on the second; a sixteenth-note suspension should occur on the first or third sixteenth note of any beat, resolving on the second or fourth; and so on.

Suspensions are labeled by the diatonic intervals they form against the bass; the main types are 2-1, 4-3, 6-5, 7-6, and 9-8.³



Sometimes the suspension can occur in the bass, in which case the labeling is tricky. The safest thing to do is to simply label all the intervals above the suspended note, as well as above the tone

² The point of this principle is that the note is in existence as a consonance longer than as a dissonance. (It goes to heaven because it's done more good than bad.)

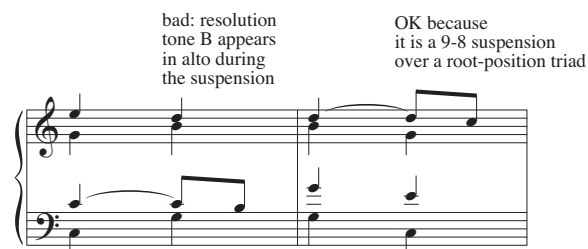
³ Note that the 6-5 is a *consonant suspension*, since the suspended note is actually consonant with the bass; nevertheless, it often acts like a true suspension. In eighteenth-century music, it can sometimes be useful to analyze and apparent first inversion chord as resulting from a 6-5 suspension, as you will learn.

of resolution. Label intervals within an octave, so that a tenth (octave plus third) is 3 and a twelfth (octave plus fifth) is 5. Ignore duplicates.

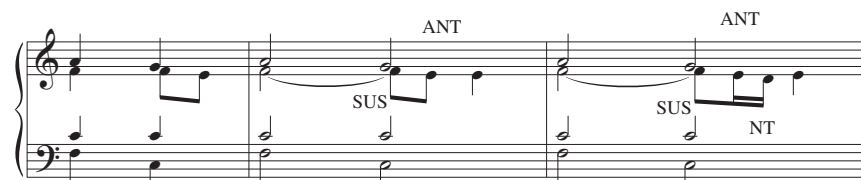


Usually, there is a third above the tone of resolution, and these bass suspensions can be abbreviated 2-3. However, if there is no third above the tone of resolution, label relative to all the other notes, as just described.

We can now state the other important rule of suspensions: *don't anticipate the tone of resolution in the other voices, except in the case of a 2-1 or 9-8 suspension over a root position triad*. That is, no other voice should double the note of resolution while the suspension is in progress. The idea here is that the tone of the resolution will clash with the suspension tone if they appear in different voices. Consequently, you want to leave room in the chord for the suspension to resolve.



Note that suspensions are sometimes reattacked, rather simply being held over. And sometimes the tone of resolution is embellished with an anticipation or an anticipation and a neighbor note. In fact, the third passage below is extremely common. For clarity, I wrote the music using slower note values, with the suspension being a quarter note long; however, the pattern also appears in faster values too.



Sometimes the chord can even change from suspension to resolution. The rule here is that the resolution tone must be a chord tone in the chord that appears under the suspension. Intuitively: if you erase the suspension, and simply replace the suspended note with the tone of the resolution, you get something that makes sense harmonically.

Chord changes as suspension resolves. The tone of resolution, C, belongs to both chords on beat 2

C: V I vi

VI. Nonharmonic tones, chord tones, and parallel fifths. Chordal skips and nonharmonic tones are not in general a means to avoid parallels. A good rule of thumb is: if you have parallels upon eliminating the nonharmonic tone (or chordal skip), then you're in trouble; the nonharmonic tone (or chordal skip) merely disguises the problem but does not make it better.

Somewhat unfairly, however, nonharmonic tones can *create* parallels. So if the soprano moves C5-D5-E5 while the tenor moves G5-G5-A5, that is parallel fifths, *even if* the D5 is a passing tone. Thus both of the following passages are bad.

passing F4 does not eliminate sop/alto parallels passing D5 creates sop/alto parallels!

There is one exception to this rule, given by the following progression.

for whatever reason, Bach allows this!

ANT

PT

C: V I

Here we have a falling fifth progression (V-I in Roman numeral terms) with scale degrees 2-1 in the soprano, and a passing tone 5-4-3 in a lower voice. For whatever reason, it appears frequently in Bach.

The Geometry of Music: two-note chords

NOTE: this handout is solely for those who are interested. You will not be tested on this material at any point. In fact, nothing bad will happen if you decide stop reading at this very instant ...

I. Ordered pitch space. You can use an ordered pair of numbers to represent the state of a two-voice ensemble: given the pair (x, y) , the first number can be taken to represent the note played by the first instrument, while the second can be taken to represent the note played by the second instrument. Figure 1 shows that there are two different ways to represent these ordered pairs geometrically. In Fig. 1a we represent them using a grey circle for the first note and a black circle for the second. (We need different colors because the pairs are ordered, and we need to differentiate the first object from the second.) In Fig. 1b the horizontal axis represents the first pitch, while the vertical axis indicates the second. It is more-or-less a matter of convenience whether we wish to represent two-note objects using two points in a one-dimensional space, or one point in a two-dimensional space.

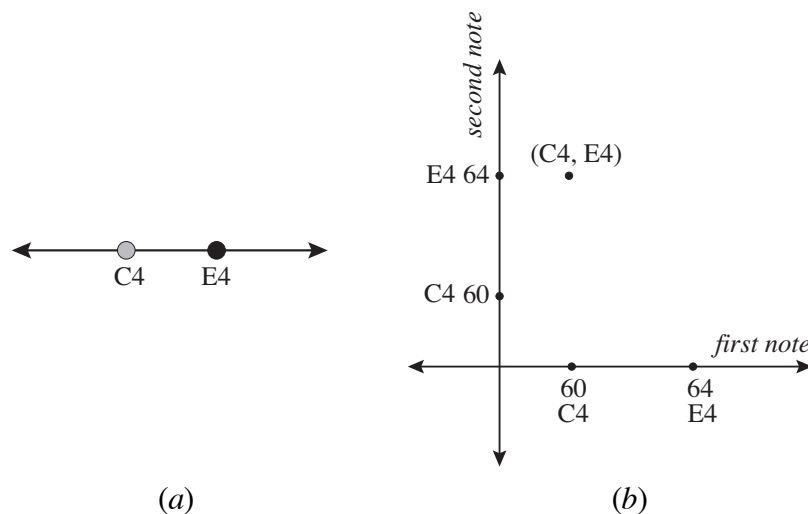


Figure 1. Two ways to represent the ordered pair $(C4, E4)$. The first uses two objects in a one-dimensional space—here, the grey circle represents the first note, and the black ball the second. The second uses a single point in a two-dimensional space.

In the two-dimensional space, progressions can be represented by line segments: thus Figure 2 represents $(C4, E4) \rightarrow (E4, C4)$ in which the bass and tenor trade notes. (Here, C4 moves to E4 and E4 moves to C4.) Figure 3 uses line segments to represent a two-voice passage from Josquin's *Missa l'homme armé*. With a little practice, it becomes easy to translate two-voice music into a series of line segments in our two-dimensional space. Any two-voice passage of music, in any style whatsoever, can be represented in this way.

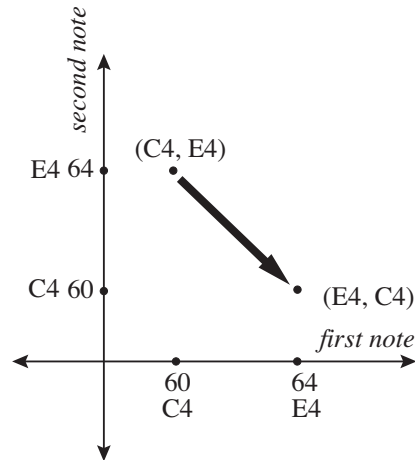


Figure 2. A voice leading can be represented as a line segment in the plane.

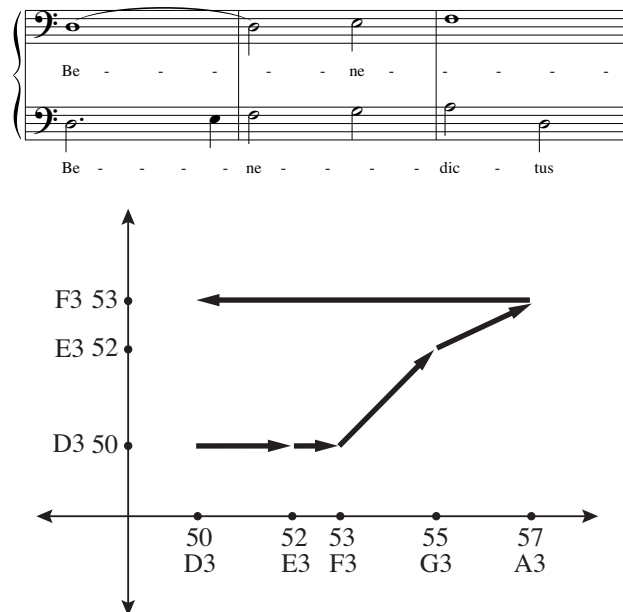


Figure 3. A passage from Josquin represented in two-dimensional space.

Horizontal and vertical line segments represent motion in a single voice (Figure 4). Parallel motion, in which the two voices move in the same direction by the same amount, occurs along the 45° NE/SW diagonal, while perfect *contrary* motion—in which the voices move the same distance in opposite directions—is represented by the 45° NW/SE diagonal. It is somewhat more convenient to rotate the space clockwise by 45° , so that parallel motion is horizontal and perfect contrary motion is vertical. The result is that chords on the same horizontal line now relate by transposition; chords on the same vertical line sum to the same value, when pitches are represented numerically. (This follows from the fact that perfect contrary motion subtracts from one voice what it adds to the other.) Oblique motion now moves along the 45° diagonals: motion the first voice occurs along the NW/SE diagonal, while motion in the second occurs along the NE/SW diagonal. Note that this rotation does not change the space in any way; we are simply changing our perspective on it.

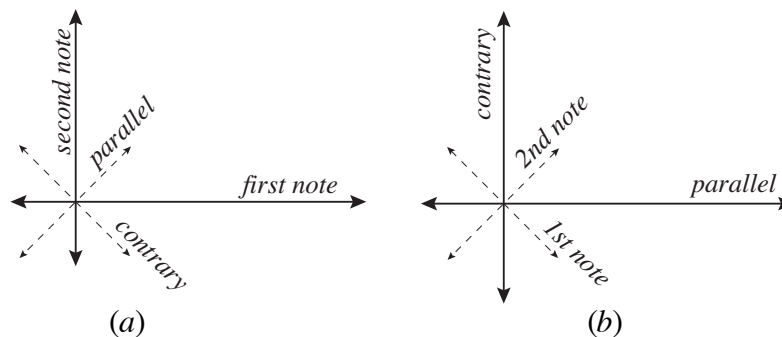


Figure 4. Rotating two-note ordered pitch space.

Figure 5 depicts an extended portion of two-dimensional ordered pitch space, with the axes rotated as just described. The most striking feature of this graph is its *periodicity*: like a piece of wallpaper, it consists in a single pattern (or “tile”) that is repeated to cover the larger plane. The figure contains four complete tiles. The points in the lower-left quadrant are related by octave transposition to the corresponding points in the upper-right: the first element in each pair is the same, while the second element in the lower-left pair is one octave below the second element in the upper-right pair. Moving from the lower-left quadrant to the corresponding point in the upper right will therefore transpose the second note up by octave. Similarly, the points in the upper left quadrant are octave-related to those in the lower right: moving from the upper-left to the corresponding point in the lower right shifts the *first* element up by an octave.



Figure 5. A portion of infinite, two-dimensional ordered pitch space.

What about the lower-left and upper-left quadrants? Here, the relationship is somewhat harder to grasp. Imagine that there were a hinge connecting the them, so that the bottom-left quadrant could be lifted out of the paper and flipped onto the upper left. This transformation maps each pair in the lower quadrant onto a pair with the same pitch content, but in the reverse order. Geometrically, the transformation is a *reflection*: a pair in the lower-left quadrant gets sent to the spot where its reflection would appear, if the common border were a mirror. When we move a dyad from the lower-left quadrant to its reflected image in the upper left, we switch the order of its notes. The two rightmost quadrants are related in the same way.

Figure 6 depicts symbolically the relationship between the four tiles, using a rightside-up human face to represent the lower-left tile. The upper-left quadrant is upside-down relative to the lower left: if they were connected by a hinge along their common border, then either could be flipped over so as to coincide with the other. Similarly, the lower-right tile is upside-down relative to the upper right. The lower-right tile is also upside-down relative to the lower-*left* tile, but in this case they cannot be related by reflection along their common border—that process would exchange left and right, but not up and down.



Figure 6. Ordered pitch space is like a piece of wallpaper.

2 *The Parable of the Ant.* Imagine now that an ant is walking along the wallpaper in Figure 6. Suppose that you and I are gambling types, and decide to bet on whether the ant will touch a pipe in the next thirty seconds. For the purpose of settling the bet, it does not matter which tile the ant is on: what matters is whether it touches *any* pipe in *any* tile. We could therefore represent the ant's trajectory on a single tile, as in Figure 7. (Suppose we want to record of our game for posterity while using the minimum amount of paper.) To do this, we arbitrarily select one tile from Figure 5, and take the ant's position to be the point on this tile corresponding to its position on the tile it actually occupies.

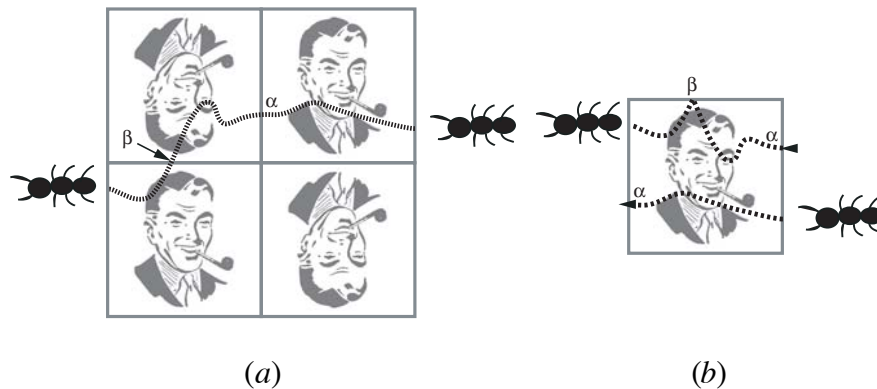


Figure 7. An ant's path can be represented on a single tile

Though the underlying idea is simple, the structure of the resulting single-tile space is complex. For example, at the point marked α , the ant disappears off the lower left edge, only to reappear on the upper right. This is reminiscent of early video games such as *Asteroids* or *Pac-Man*, where objects could move off one side of the screen to reappear on the other; unlike those games, however, the ant leaves the *lower* half of the figure only to reappear on the *upper* half, and vice versa. It is as if the left edge were attached to the right, but with a twist. The mathematical name for such a space is a *Möbius strip*.

Now consider point β in the ant's trajectory. In the single-tile representation the ant appears to "bounce off" the figure's upper edge: it is as if the top and bottom edges acted as mirrors, or the bumpers on a pool table, reflecting the ant backward away from the boundary. But we can see from Figure 7a that the ant's actual trajectory is straight. Nothing intrinsic to the ant's motion produces the change in direction—rather, it is the structure of the wallpaper on which it walks. The ant, being unaware of the wallpaper's pattern, would have no knowledge that anything unusual was occurring.

All this talk about ants, wallpaper, and gambling may seem like a distraction from the serious goal of understanding music. But in fact, we can form a precisely similar structure by "folding up" the infinite two-dimensional *musical* space in Figure 5. In this case, ignoring what tile we are on corresponds to ignoring the *order* and *octave* of a pair of notes. In other words, the resulting space represents two-note chords, as musicians are accustomed to thinking of them. Thus the Parable of the Ant, rather than being a frivolous digression, has actually marked the beginning of our investigation into a remarkable convergence between music theory and contemporary geometry.

3 Two-note chord space. Our goal is to "fold" the infinite space of Figure 2 so as to glue together all the different points representing the same chord: (C4, E4), (E4, C4), (E5, C2), and so on. The result will be a new geometrical space, analogous to our single tile of wallpaper, in which points represent *chords*—or unordered pairs of pitch classes—rather than ordered pairs of pitches.

Happily, the work has already been done. The "wallpaper space" described in the preceding section was modeled on Figure 5, and we can therefore immediately adapt our results to the musical case. Since each of the quadrants on Figure 5 contains precisely one point for every unordered set of pitch classes, any of them can be used to represent two-voice music. Figure 8 depicts a single quadrant of Figure 5, removing octave designations to reflect the fact that we have discarded octave information. Though the points are labeled using ordered pairs, this ordering is not significant: the point $CD\flat$ represents the ordering ($D\flat$, C) just as much as it represents the ordering (C, $D\flat$).

In §1, we rotated the coordinate axes, so that parallel musical motion is represented by horizontal geometrical motion. It follows that chords on the same horizontal line of Figure 8 are related by transposition: the top edge of the figure, for example, contains unisons—two note chords in which both voices have the same pitch class. Unisons can also be found on the bottom edge of the figure, with two of them—{C, C} and {F $\sharp$, F $\sharp$ }—appearing on both edges. (Chords on the right side of the figure are enclosed in brackets, indicating that they also appear on the left side.) These duplications are different representations of the same chord, artifacts of the doomed attempt to depict a Möbius strip in two Euclidean dimensions.

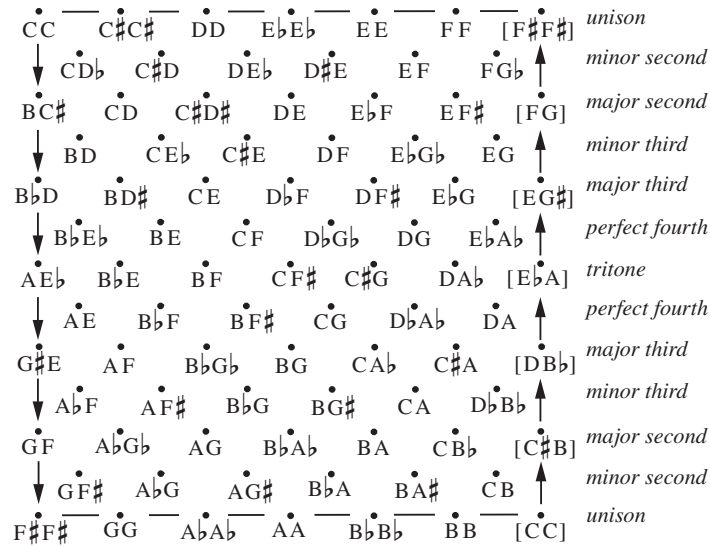


Figure 8. Two-note chord space. The left edge is “glued” to the right, with a twist.

The other intervals are also found on horizontal line segments, one in the top half of the figure and one in the bottom. For example, the horizontal line-segment just below the top edge, and just above the bottom edge, contains minor seconds. (Again, these line segments are attached at their endpoints, and therefore form two halves of an abstract circle.) Major seconds are just below the minor seconds on the top half, and just above the minor seconds on the bottom. As the interval between the two notes gets larger, dividing the octave more evenly, the lines representing them move toward the center. Consequently, the line at the very center of the figure contains tritones, which divide the pitch-class circle into two halves.

Chords on the same vertical line can be linked by exact contrary motion: for instance, the voice leading $(C, F\sharp) \rightarrow (D\flat, F)$ moves C up by semitone to $D\flat$, and $F\sharp$ down by semitone to F . Consequently, every dyad lies on the same vertical line as its tritone transposition.¹ Finally, as we would expect, motion along the 45° diagonals moves just one note in the dyad: thus $(C\sharp, E)$ lies 45° northeast of (C, E) . Once again, any two-voice passage of music, in any style whatsoever, can be represented in this space. Every point in the space, represents a single chord, while every line segment represents a *voice leading*, or a way of mapping the notes of one chord to those of another.

¹ One can always connect tritone-related dyads by contrary motion: given the dyad $\{C, E\}$, one can move C up by tritone to $F\sharp$, and E down by tritone to $A\sharp$.

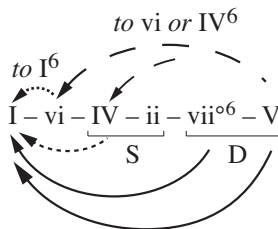
STYLE 3: FUNCTIONAL HARMONY

Style 3 (baroque/classical/romantic = “common practice”) adds to Style 2 (Renaissance) a new set of specifically *harmonic* rules about which chords can follow which other chords. For this reason, Style 3 is said to involve “*functional harmony*.” The term implies that each chord has its own particular function, or role.

I. Scales. The music will be in either the major or minor mode. In the major mode, use triads belonging to a single diatonic scale. In minor mode, *harmonic minor* is the default scale! Melodic minor will be used only to avoid the harmonic minor scale’s augmented second.

II. Rhythm, Voices and Ranges, Counterpoint, Harmony. All of the rules in sections II-V of the Style 2 handout remain in effect. You can (should) continue to use the voice leading strategies discussed earlier (“open,” “half-open,” “close,” etc.).

III. Basic functional harmonic chord progressions. The following map shows the allowable chord progressions in major:

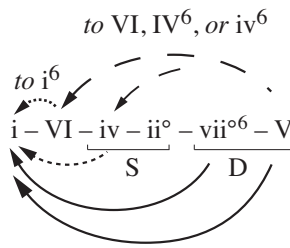


Any chord can move rightward along this map by *any amount*: I can move rightward to vi, IV, ii, vii°6, or V; vi can move to IV, ii, vii°6, or V. Chords can move leftwards only along the arrows.

The “I” chord is the tonic chord: it is the most stable, “home” chord. Phrases typically begin and end with a **root position** I chord. The vii°6 and V chords are “dominant” chords: they sound tense and expectant, and they “want” to move to dominant chords. The IV and ii chords are “subdominant” or “predominant” chords; they tend to lead toward dominant chords. Note that iii is not used, and that neither iii nor vi have their functional names (like “tonic,” “dominant” or “subdominant”).

The progressions represented by the large-dash line, $V \rightarrow vi$ and $V \rightarrow IV^6$, are called *deceptive progressions*. The $IV \rightarrow I$ progression is called a *plagal progression*.

It should be emphasized that these maps give only a first approximation to the chord progressions of elementary tonal harmony. There are a few other idiomatic chord progressions you will need to learn if you want to become truly fluent in this language. However, this handout gives you a pretty good start: more than 95% of the chord progressions in a typical tonal piece use the moves depicted on the above maps. (As in other areas of life, getting 95% of the way toward expertise is relatively easy; it’s the last 5% that really requires work!)



The map for minor key music is essentially the same. Note that we typically use the triads corresponding to the *harmonic minor* scale. **Consequently, the seventh scale degree will always need to be raised by semitone with an accidental!** Note also that the ii chord is now diminished: **it can therefore appear only in first inversion.** Use of the melodic minor sometimes creates a minor ii chord or a major IV chord. These chords continue to function as shown on our map.

IV. Inversions. Chords typically appear in root position, which is about twice as likely as first inversion. However, the ii chord (in both major and minor) typically appears in first inversion rather than root position. The vi chord appears **only** in root position: vi⁶ is basically unused. This means that only one triad can appear above scale degrees 1, 3, 5, and 7.

A tonal phrase consists in a number of cycles through the above graphs. Each cycle creates increasing tension: the stable tonic leads to the (unstable and tense) dominant chords, and then back again.¹ Composers therefore face a problem: how do you keep these cycles from sounding repetitive and boring? How do you make the end of the phrase sound conclusive, while avoiding premature conclusiveness during the earlier V-I progressions?

There are two main techniques, each of which involves the outer voices. In the opening portions of the phrase, one often uses *inversions* to weaken the force of the earlier V-I progressions, saving the strongest root position V-I progression for the end of the phrase. (This is not to say that one can *never* use root position V-I progressions earlier in the phrase; they're used sometimes.) Similarly, one can make sure that the melodic voice in the earlier V-I progressions involves the third or fifth scale degree over the I chord, saving the melodic tonic note for the end of the phrase.

The following example exhibits both techniques: the bass uses inversions, leading to a strong, root position V-I cadence; the soprano moves stepwise to a 2-1 close at the end of the phrase.

¹ Each cycle can be compared to an English sentence, with its Subject-Verb-Object ordering; a series of cycles, forming a phrase, can be compared to a paragraph, which is a series of English language sentences.

Melodically, classical phrases seem to fall into two broad categories: *arches*, which rise to a high point and then fall, usually to the first scale degree, and *descents*, which begin at a high point and then fall to the first scale degree.

V. The leading tone. In functional harmonic music, the leading tone is a very active tone that “wants” to move to the tonic. For this reason, you have to observe two new rules concerning the leading tone. 1) In a dominant chord (V and vii^{o6}), the leading tone should *never* be doubled. 2) The leading tone will normally move *upward* by semitone to the tonic note. *Note that these rules apply not just to V→I and vii^o→I progressions, but also to V→vi and V→IV⁶ “deceptive” progressions!*

This is particularly important when the leading tone is in one of the outer voices. When the leading tone is in an inner voice, it will occasionally move down—particularly if another voice “covers” this fact by sounding the note that the leading tone wanted to resolve to.

If the leading tone moves downward, don’t advertise this fact by adding a passing tone!

V I V I V I V I

good: leading tone resolves up by step

good: leading tone moves down, but is covered by an upper voice, which provides the right resolution

bad: if the l.t. doesn't resolve right, don't draw attention to this with a passing tone!

bad: the leading tone in an outer voice resolves incorrectly

VI. Seventh chords. Classical music revolves around the journey from stable tonic to unstable dominant and back. In order to heighten this contrast, the dominant chord often appears as a *seventh chord*: for instance, in C major, the dominant seventh G-B-D-F is about twice as common as the bare triad G-B-D. This gives it more of a dissonant “oomph.”

The seventh is a dissonance which must resolve *down by step*. In general, you should not leap into the seventh. Seventh chords can appear in any inversion, and typically none of their notes are doubled. However, you can occasionally omit the fifth of a root-position V⁷ chord, doubling the root.

The next most common seventh chord is the ii⁷, which typically appears in first inversion, as a ii⁶ chord. (About 10% of ii chords are sevenths.) Seventh chords on vii^o are not unknown. On all other chords they are rare, at least in chorale style.

See the earlier handout on Roman Numeral Analysis for more information on sevenths.

VII. Phrase beginnings and endings. Phrases should generally begin with a *root position tonic chord*. Phrases typically end with a V→I progression in which *both chords are in root position*. This is called an “authentic cadence.” You can also end with one of the other cadences listed below.

Progression (major)	Progression (minor)	Name	Comments
V→I	V→i	authentic cadence	melody ends on scale degree 1: perfect authentic cadence
			melody ends on s.d. 3 or 5: imperfect authentic cadence
I→V (more rarely: IV→V or ii ⁶ →V)	i→V (more rarely: iv→V or ii ⁶ →V)	half cadence	sounds incomplete; comma rather than period
	iv ⁶ →V	phrygian cadence	
V→vi	V→VI	deceptive cadence	trick ending: scale degrees 2 and 7 converge on 1
IV→I	iv→i	plagal cadence	weak conclusion; “amen”

Cadences are like musical punctuation marks. The authentic cadence (V→I) is a period. The half-cadence, which ends on V, is a comma, since it sounds incomplete. (In minor, the half cadence iv⁶→V is called a phrygian cadence.) The deceptive cadence (V→vi) is surprising; perhaps it is like a question mark. To achieve the appropriate level of surprise, scale degree 2 typically moves down by step to the tonic, while the leading tone moves up by step to the tonic. (This is the medieval “converging voices” cadence, still with us, though in a weakened form, several centuries later!) The plagal cadence (IV→I) sounds archaic and severe—it is associated with the final “Amen” in many religious pieces.

Note that you no longer have to obey the Renaissance/medieval convention whereby two voices converge at the cadence; the new harmonic formulae have taken over the burden of articulating the cadence.

VII. How to write functional harmony. Suppose you are given a melody and asked to harmonize it. What do you do?

Here’s a simple three-step procedure.

1. For each melody note, write down the acceptable harmonies containing it.
2. Find a preliminary path through these harmonies that obeys the principles of the map shown in §3.
3. Construct a bass line that goes with these harmonies. Make sure it starts with a root position chord and ends with a root-position cadence. Double check to make sure the bass line does not make parallel fifths and octaves with the melody. *You may find*

yourself altering the preliminary path you chose at stage 2, depending on how the bass line develops.

4. Write inner voices to go with the outer two. *You may find yourself altering the bass line, or the harmonic progression, to ensure smooth inner voices. Usually, however, you can make the inner voices work.*

The procedure is essentially similar if you're given a bass line and asked to harmonize it. Steps 1 and 2 are the same; at step 3 you write down a melody rather than a bass line.

VIII. Checklist. Once you've completed a harmonization, you should go through the following checklist. You can find your mistakes as well as we can—and it's much better for your grade if you do it than if we do!

1. Write Roman numerals (with figured bass numbers) under each of your chords.
2. Check your chord progressions to make sure the chord progression is legal.
3. Check for illegal inversions:
 - no $\frac{6}{4}$ chords
 - vi^6 should not be used
 - diminished triads (vii° and ii°) only in first inversion
4. Check to make sure you haven't doubled the leading tone in V and vii^{o6}
5. Did you resolve the leading tone upward, except as discussed in section V?
6. Did you make sure you raised the leading tone in minor?
7. Watch out for parallel fifths and octaves! Check every pair of voices individually!

Here is a statistical picture of the Bach chorales and the Mozart Piano Sonatas. For Bach, I studied the first 70 chorales in your Riemenschneider volume. For Mozart, I looked at analyses of all the piano sonatas, containing about 13,000 chord progressions.

Note that I, IV, and V account for about 50-75% of the chords in this music. Use them often!

I. Chord Distributions

Bach major

	% of total	% root	% first	% second	% seventh
I	25	60	35	5	0
ii	9	77	22	1	53
iii	2	76	24	0	7
IV	12	54	46	0	8
V	35	72	28	1	42
vi	6	92	5	3	7
vii	7	10	88	2	24

Mozart major

	% of total	% root	% first	% second	% seventh
I	41	53	23	23	0
ii	9	30	69	1	11
iii	0	100	0	0	0
IV	8	60	23	17	0
V	34	74	23	3	67
vi	3	95	5	1	12
vii	3	50	39	11	12

The second column shows each chord as a percentage of the total: e.g. 25% of all chords in Bach are tonic chords. The next columns shows the likelihood that this chord is in a particular inversion: e.g. in Bach, 60% of all tonic triads are in first inversion. The last column shows the likelihood that the chord will be a seventh: e.g. in Bach, 0% of tonic triads are sevenths. Note that in Mozart (but not Bach!) ii is more likely to occur in first inversion; vi⁶ is almost completely absent from both composers. I⁴ and IV⁴ chords are very popular in Mozart; you'll learn about them soon.

II. Probabilities for the various chord progressions

The first chord is on the left, and the second chord is above. The number in the cell indicates the percent chance that the chord on the left will move to the chord on the top. Overall, the progressions fit our model very nicely: more than 90% of the vi→I progressions are to I⁶, and more than 90% of the V→IV and vii⁶→IV progressions are to IV⁶.

Bach major

	I	vi	IV	ii	vii	V	iii
I	*	9	28	15	6	41	1
vi	12	*	11	30	9	33	5
IV	22	2	*	13	23	39	0
ii	1	1	0	*	25	71	0
vii	91	3	2	0	*	4	1
V	82	9	7	1	0	*	0
iii	3	32	52	3	3	6	*

Mozart major

	I	vi	IV	ii	vii	V	iii
I	*	5	15	13	5	62	0
vi	9	*	14	52	4	21	0
IV	50	0	*	19	10	21	0
ii	1	1	1	*	18	77	0
vii	82	0	1	0	*	16	0
V	94	4	1	0	1	*	0
iii	67	33	0	0	0	0	*

III. The most popular harmonic cycles (progressions starting and ending with the tonic)

Bach

I-V-I	90
I-ii-V-I	30
I-IV-V-I	26
I-IV-I	22
I-vii°-I	21
I-ii-vii°-I	13
I-IV-ii-V-I	9
I-IV-vii°-I	7
I-vi-ii-V-I	5
I-V-IV ⁶ -V-I	4

Mozart

I-V-I	1026
I-ii-I ₄ ⁶ -V-I	204
I-vii°-I	127
I-IV-I	122
I-ii-V-I	82
I-IV-I ₄ ⁶ -V-I	68
I-vi-ii-V-I	61
I-I ₄ ⁶ -V-I	60
I-ii-vii°-I	27
I-V/IV-IV-I	23
I-IV-V-I	20
I-V/IV-IV-vii°-I	19
I-IV ⁶ -I ₄ ⁶ -V-I	18
I-vi-IV-ii-V-I	17
I-V/ii-ii-V-I	15
I-V-vi-V-I	15

(You'll learn about V/IV and V/ii chords later.)

MUSIC 105

Dmitri Tymoczko
Handout 15 (2010)

THE CADENTIAL I $\sharp$

As you know, the I $\sharp$ chord is not generally used in either Renaissance or baroque/classical/romantic music. However, there are a few exceptions to this general rule. The most important is the *cadential* I $\sharp$ chord, which usually precedes a root-position V and helps to signal the end of a phrase. (The root position V, in turn, often moves to a V⁷ by way of passing motion.) The cadential I $\sharp$ creates a strange revaluation of tonal values: it is a *dissonant* tonic triad that serves to push the music toward the dominant chord, which itself wants to resolve to a (root position) tonic. This involves a kind of double dissonance: a dissonant chord that leads to another less dissonant chord that itself leads to the tonic.

Since the fourth above the bass (i.e. the root of the chord) is dissonant, it needs to be treated carefully: in fact, it *resolves down by step* just like a suspension or chordal seventh. For this same reason, the root is never doubled. Typically the third of the chord (the sixth above the bass) also resolves down by step. This means you have to double the fifth of the chord, the bass of the I $\sharp$. One way to remember this is that the three upper voices should sound a complete triad, moving in the most natural way to a complete dominant chord: that is, the root and third should move down by step, with the fifth being held over as a common tone.



The other important rule involves rhythm. Like the suspension, the I $\sharp$ must be *rhythmically stronger* than the V chord that it resolves to. For example, the chord can appear:

- On beat 1 of a 4/4 measure, resolving on beats 2, 3, or 4.
- On beat 3 of a 4/4 measure, resolving on beat 4.
- On beat 1 or 2 of a 3/4 measure, resolving on beat 3.
- On any strong eighth note, resolving on a weak eighth note.

Finally, like a suspension, the I_4^6 chord should be *prepared*. As with seventh chords, the rules are a little more flexible than in the case of the suspension. Here the basic idea is that you should approach the root and third of the chord by step or by common tone, and not by leap.



In chorale style, the I is almost always preceded by IV or ii^6 . In later instrumental music (Haydn, Mozart, etc.), the rules are more flexible, and the chord can be preceded by I , vi , IV , or ii (but not vii°). In classical instrumental music, it is very common to move repeatedly back and forth between V and I_4^6 .

It is important to understand that the I_4^6 chord is closely related to the double suspension: it is almost as if the root and third were “nonharmonic tones” since they must resolve downward by step, since they must be rhythmically strong, and since they must be prepared (although the rules for preparation are more flexible in the case of I_4^6 than in the case of a true suspension). In fact, some theorists reflect this in their notation by labeling the chord V_{4-3}^{6-5} instead of I_4^6 . This is supposed to mean something like “a V chord with a 6-4 suspension and a 4-3 suspension occurring simultaneously.” However, I prefer to label the chord I_4^6 , since I think it is a genuinely harmonic object rather than a true suspension.

Here’s a little checklist to make sure you’re using the chord correctly:

- double the bass
- resolve the root and third down by step
- the chord must be rhythmically stronger than the V it resolves to.
- approach the root and third by step or common tone.
- in chorale style, precede with IV or ii^6 .
- it’s common to have a descending octave leap in the bass.

Note that in minor keys the chord behaves just as you would expect: it resolves to a major V (with raised leading tone) and it is preceded by iv or $ii^{\circ 6}$.

APPLIED DOMINANTS

Consider the following chord progressions:

C: I V_5^6/ii ii V^7 I B $\flat$: i V^7/V V ⁽⁷⁾ i

What's new about these progressions is that we can no longer attribute all the chords to a single key. The A^7 in progression (a) belongs to d minor, but the progression as a whole belongs to C major. Likewise, the C^7 in progression (b) belongs to F, but the progression as a whole is in the key of B $\flat$ minor.

These chords are called *applied* or *secondary* dominants. *Any major or minor triad in a key can be preceded by its own dominant chord.* It is as if the music shifts key for a very brief time, making a subsidiary triad into a temporary tonic. (This is called “tonicization”: the above examples tonicize D minor and F major, respectively.) Another way to describe the phenomenon is that a dominant chord is “borrowed” from a closely related key. The result of a secondary dominant is to create a hierarchically nested series of tonalities: the local tonality (D minor in the first example) is embedded in a larger tonality (C major, in this case). This is similar to the way an English clause can be embedded in a larger clause: “the boy who got bitten last week kicked the dog.”

Secondary dominants add a lot of musical color, and help create a strong sense of harmonic motion. It is possible to overuse them. But classical (and jazz) harmony typically makes very frequent use of applied dominant chords. In fact, V^7/V is one of Beethoven's favorite chords—it appears about as often as ii or IV.

Here are some basic principles governing the use of these chords.

1. The borrowed chord is usually always a dominant seventh chord. Borrowing a major triad is less common – the added dissonance of the seventh chord makes it want to lead to the next chord all the more strongly. (In rock music, however, it is quite typical to borrow a major triad rather than a seventh chord.) It is also possible to borrow a leading tone triad ($vii^{\circ 6}$) or seventh chord.

2. Normal voice-leading rules apply to the borrowed dominant chord as if you had changed key. **You cannot double the leading tone of the applied dominant.** You should resolve the borrowed chord's tendency tones normally. (But see the exception listed as point 8 below.) You should also spell the chord correctly, as dominant of the chord to which it resolves.

3. The applied dominant almost always resolves to the appropriate temporary tonic chord, but it can occasionally participate in a deceptive progression, moving up by step. For example, you might find V/vi moving to IV (aka VI/iv) rather than vi.

4. *Beware the cross-relation!* A “cross-relation” is a clash between two different forms of a single note in consecutive chords in different registers. (Refer back to the Style 1 handout!) In the first example below, the soprano's high E $\flat$ conflicts with the bass E $\sharp$ in the next chord. This was felt to be an undesirable sort of dissonance. In (b) there is no cross relation, even though the high E $\flat$ still contrasts with the low E $\flat$. The reason is because the high E $\flat$ is supported by a low E $\flat$ in the bass. The bass E $\flat$ neutralizes the cross-relation since it moves to E $\flat$ in the same register, and keeps us from noticing the sharp dissonance between soprano and bass.

The image shows two musical examples side-by-side. The first, labeled 'cross-relation (bad)', shows a progression from a chord with a high E $\flat$ in the soprano to a chord with a low E $\sharp$ in the bass. A line connects the high E $\flat$ in the first chord to the low E $\sharp$ in the second chord, illustrating the clash. The second, labeled 'no cross-relation (good)', shows a progression from a chord with a high E $\flat$ in the soprano to a chord with a low E $\flat$ in the bass. The high E $\flat$ in the first chord is supported by a low E $\flat$ in the bass of the same chord, and the bass E $\flat$ moves to E $\flat$ in the same register in the second chord, neutralizing the cross-relation.

Cross-relations are not always bad. Classical composers sometimes used them to add expressive “bite,” or pungent dissonance, to their harmony. You should probably avoid them however, unless it's really clear that you know what you're doing.

5. In root position, don't move from an unaltered version of a chord to the borrowed version of the chord. A lot of students tend to write banal progressions like (a) below. This is not very stylistic. The progression from ii to V/V sounds clunky and obvious. Here it would be better just to move from I to V/V.

The image shows two musical examples side-by-side. The first, labeled '(a) banal', shows a progression from a chord with a high E $\flat$ in the soprano to a chord with a low E $\sharp$ in the bass. The second, labeled '(b) sophisticated!', shows a progression from a chord with a high E $\flat$ in the soprano to a chord with a low E $\flat$ in the bass. The high E $\flat$ in the first chord is supported by a low E $\flat$ in the bass of the same chord, and the bass E $\flat$ moves to E $\flat$ in the same register in the second chord, neutralizing the cross-relation.

Note that the progression in (b) is OK. This is because the dramatic change in the bass tones adds a lot of energy to the progression. For the same reason, you can move from I to V²/IV, since there is a powerful change of bass in the progression.

6. You cannot apply a dominant seventh chord to a diminished or augmented triad, since these triads cannot serve as “tonics,” even temporary ones.¹ This means you cannot use V/vii° in major, V/ii° in minor, or V/vii° in minor. In minor, it is also unusual to use secondary dominants of the chords involving the raised sixth scale degree: V/ii and V/IV.

Here are the normal chords which can support applied dominants.

MAJOR	MINOR
ii	-
iii	III (note that V/III is just VII)
IV	iv
V	V or v
vi	VI
-	VII (i.e. the major chord based on the <i>natural</i> seventh degree)

7. When a secondary dominant moves to another dominant seventh chord, the leading tone often does *not* resolve by step. Instead, it becomes the *seventh* of the following chord, as in the following progression:



Sometimes one finds long chains of applied dominants, as in the Mozartian chord progression below. Here, the leading tone is always lowered semitonally, creating a descending chromatic line.

C B⁷ E⁷ A⁷ D⁷ G⁷ C
 I V⁷/V/V/V/V V⁷/V/V/V V⁷/V/V V⁷/V V⁷ I

8. The V⁶/V chord can come either before or after a I⁴. In the progression V⁶/V → I⁴ → V, the I⁴ is inserted between the V⁶ and its resolution. In the progression I⁴ → V⁶/V → V, the V⁶/V is inserted between I⁴ and its resolution. Both of these work.

9. Occasionally an applied chord will be preceded by its own predominant, as in the progressions I → iv⁶/ii → V⁷/ii → ii → V⁷ → I.

¹ There is, so far as I know, only one possible exception in the Bach chorales: chorale 89, mm. 5.

MUSIC 105

Dmitri Tymoczko
Handout 17 (2010)

MODULATION IN FUNCTIONALLY TONAL MUSIC

In the Renaissance, musical phrases could end with modal cadences on various scale degrees. For example a C ionian piece might contain cadences on D (suggesting D dorian), E (suggesting E Phrygian), G (suggesting G mixolydian), and so on. Thus to create harmonic variety, Renaissance music moved around *within a single diatonic scale*.

By contrast, in functionally tonal music (aka Style 3), only two modes—major and minor—are used. To create long-term harmonic variety, the music needs to change keys or *modulate*. For instance, instead of moving from C ionian to G mixolydian, the music would move from *C major* to *G major*. Rather than shifting around within a single diatonic scale we move from one diatonic scale to another.

I. Pivot chords. The smoothest way to modulate is by way of a *pivot chord* that belongs to both the old and new key. Pivot chords can create very gentle modulations, since you cannot tell precisely where the old key ends and the new key begins. Here is a very common example:

$$\begin{array}{ccccc} \text{C} & \text{b}^{\circ 6} & \text{C} & & \text{a} & \text{D}^7 & \text{G} \\ \text{C major: I} & & \text{vii}^{\circ 6} & \text{I} & & \text{vi} & \\ & & & & \text{G major: ii} & \text{V}^7 & \text{I} \end{array}$$

The A minor chord belongs to both C major and G major, as a result the modulation is extremely smooth. Note that all the chord progressions are legal in one of the two keys.

Once you reach the pivot chord, it is a good idea to move as quickly as possible to a V⁷-I progression in the new key. The seventh is crucial here, as it forms a decisive signal that we are moving to a new key.¹

The best pivot chords are chords that are *predominant chords* (ii or IV) in the new key. That is because these chords lead smoothly to the dominant chord in the new key. In major, there are pivot chords that are predominants of iii, V, and vi (see the table below). In minor, there are pivot chords that are predominants of III and v.

Note that modulations to V or iii involve *adding a sharp* (or natural) to one of the notes in the scale. Such modulations are often called *sharp-side* modulations, since they move sharpwards on the circle of fifths.

¹ While the triadic chord progression G-C belongs to several keys (for instance, G major and C major), the chord progression G⁷-C belongs only to C major.

II. Direct modulations. What about modulations that require us to add a flat? For instance, suppose I want to modulate from C major to F major? Or from C major to D minor? In these cases, pivot chords are less common. This is because the predominant chords in the new key do *not* belong to the old key; hence it is harder to find a pivot chord that leads naturally to the V^7 in the new key.

Here we use an even simpler solution: we simply move from the tonic directly to V^7 in the new key. For example, to modulate from I to IV, just add a seventh to the I chord, converting it into the V^7 of IV.

$$\begin{array}{ccc} \mathbf{C} & \mathbf{b^{\circ 6}} & \mathbf{C^6_5} \quad \mathbf{F} \quad \mathbf{C^7} \quad \mathbf{F} \\ \text{C major: I} & \text{vii}^{\circ 6} & \\ & \text{F major: } V^6_5 \quad \text{I} \quad V^7 \quad \text{I} \end{array}$$

Note that in this progression, there is no pivot chord. Again, this is because ii and IV in F major (G minor and $B\flat$ major) do not belong to C major.

The remarks in the preceding paragraph also apply to modulations from i to iv in minor—pivot chords are not always used in these cases.

$$\begin{array}{ccc} \mathbf{c} & \mathbf{b^{\circ 6}} & \mathbf{C^6_5} \quad \mathbf{f} \quad \mathbf{C^7} \quad \mathbf{f} \\ \text{C minor: i} & \text{vii}^{\circ 6} & \\ & \text{F minor: } V^6_5 \quad \text{i} \quad V^7 \quad \text{i} \end{array}$$

Similarly, when modulating from I to ii in major, one often dispenses with the pivot chord:

$$\begin{array}{ccc} \mathbf{C} & & \mathbf{A^6_5} \quad \mathbf{d} \quad \mathbf{A^7} \quad \mathbf{d} \\ \text{C major: I} & & \\ & & \text{D minor: } V^6_5 \quad \text{i} \quad V^7 \quad \text{i} \end{array}$$

Note that one never modulates to the supertonic in minor, since the standard minor ii chord is diminished, and cannot be the tonic of either a major or minor key.

You can use this strategy when modulating to any key whatsoever: even though there is a pivot chord available, you do not always have to use it. Thus composers often modulate to the dominant by going directly to V^7 of the new key, bypassing the pivot.

III. Standard modulatory routes. Most tonal pieces follow a reasonably standard modulatory pattern. In major, the first modulatory destination is almost always the dominant key. After that, one can find a variety of minor keys. The subdominant tends to appear near the end of the piece, as we prepare to return to the tonic. In minor,

the first modulatory destination is almost always the relative major. (Sometimes one can modulate to the minor dominant instead; one almost never modulates to the major dominant.) As in minor, the subdominant key tends to appear near the end of the piece.

MAJOR:

Beginning	First Key	Second Key	Other Keys	Penultimate Key (optional)	End
I	V	ii (or vi)	various	IV	I

MINOR:

Beginning	First Key	Second Key	Other Keys	Penultimate Key (optional)	End
i	III (or v)	VII, iv, VI	various	iv	i

IV. Tonicization of III in minor. Within minor-key phrases, one often finds very brief trips to the relative major, III. For example, chord progressions such as the following are common.

	c	B$\flat$	E$\flat$	f	G^7	c
C minor:	i			iv	V 7	i
E $\flat$ major:	vi	V	I	ii		

Here, the progression B $\flat$ →E $\flat$ sounds like V-I in E $\flat$ major; however, rather than staying in this new key, the music immediately returns to the home key of C minor. (Usually, the music returns to the home key by way of a predominant in the home key—here F minor or the D diminished triad, ii and vii $^\circ$ of E $\flat$, and iv and ii of C.) Such brief detours (including progressions using applied dominants) are usually called *tonicizations* rather than *modulations*. “Tonicizations” are very short trips to other keys, often just 1-2 chords long; “modulation” implies a longer stay in the new key. You should try to master the technique of switching briefly to the relative major in your minor-key harmonizations.

IV. Common pivot chords. Here are the most common pivot chords, for the most common modulatory destinations in major and minor. In general, the best pivot chords are predominant chords (ii or IV) in the new key.

MAJOR

Modulatory Destination	Common Pivot Chords (Best to worst)
ii	pivot chord not needed ii (old key) = i (new key)
iii	vi (old key) = iv (new key) I (old key) = VI (new key)
IV	pivot chord not needed IV (old key) = I (new key) I (old key) = V (new key) ii (old key) = vi (new key)
V	vi (old key) = ii (new key) I (old key) = IV (new key) V (old key) = I (new key)
vi	ii (old key) = iv (new key) IV (old key) = VI (new key) vi (old key) = i (new key)

Note that it is very uncommon to modulate to vii, since vii is a diminished triad.

MINOR

Modulatory Destination	Common Pivot Chords (Best to worst)
III	ii ^{o6} (old key) = vii ^{o6} (new key) iv (old key) = ii (new key) VI (old key) = IV (new key)
iv	pivot chord not needed iv (old key) = i (new key)
v	i (old key) = iv (new key)
VI	iv (old key) = vi (new key)

*Note that it is uncommon to modulate to ii in minor, since the normal form of ii is a diminished triad, which cannot be the tonic of a key. Furthermore, one always modulates to the **minor dominant key** (v) rather than the **major dominant key** (V).*

MUSIC 105

Dmitri Tymoczko
Handout 18 (2010)

STYLE 4: CLASSICAL KEYBOARD MUSIC

Style 4, you'll be happy to learn, adds no new harmonic or voice-leading rules to those of Style 3. The differences lie elsewhere: instrumentation, rhythm, texture, and expressive intent. First, we'll be writing keyboard rather than vocal music. Second, this style makes a significant distinction between the *melodic* voice and the accompanimental voices. (In Style 3 vocal music, voices are closer to being equal. In Style 4, the top voice predominates over the others.) Third, we'll be writing dramatic music that values significant changes in texture and mood. By contrast, Style 3 music (indeed Baroque music generally) values consistency over dramatic change.

I. Ranges. The music should be written on two staves for a normal, two-handed, 10-fingered pianist. The right hand plays the melody, and typically ranges from C4 to about C6, or even F6. (**The melody normally occupies the area around C5.**) The left hand plays the accompaniment. It typically occupies the tenor vocal range, from C3 to G4. **Do not write chords below C3;** these will sound muddy. It is perfectly normal for the accompaniment to occupy the range around middle C. **NB:** well-mannered classical composers do not ask normal, two-handed, 10-fingered pianists to try to reach a distance of greater than an octave with one hand.

II. Rhythm. The music should be written in 2/4, 3/4, 4/4 or 6/8, in a reasonably quick tempo. Quarter notes and eighth notes predominate in the melody. (For now, relatively few sixteenths, except in Alberti bass patterns [see below]!) Half notes, triplets, rests, etc., are permitted. Cadences should occur on the strong beats of the measure—that is, beat 1 in 2/4 and 3/4, and beats 1 or 3 in 4/4. *Chords change, on average, once per measure.* However, they can sometimes change more quickly. This is particularly common near the ends of phrases, where the harmonic rhythm often accelerates. Occasionally, one will find the same chord in two successive measures—usually with a change in inversion, however.

The accompanimental voices often move in the same rhythm. The melodic voice tends to have its own rhythm, distinct from the accompaniment. For this reason, the music often feels like it is in two parts (melody and accompaniment), even though the accompaniment may technically contain several voices.

Phrases tend to be eight bars long. This norm becomes more robust as time goes on: in Haydn, eight-bar phrases are relatively rare; in Mozart they happen more frequently; and in Beethoven they are ubiquitous. You should use them frequently. Note

the following rhythmic subtlety: eight bar phrases can either cadence in bar 8 or on the first beat of bar 9, overlapping with the start of the next phrase.

III. Number of voices. The music is often written for three voices—one melodic voice, a bass line, and an inner voice which plays in the same rhythm as either melody or bass. However, the number of voices can change. You are allowed brief passages of two voice writing, and brief passages of four-voice writing. (It is typical for the harmony to thicken near a cadence.) Changes should occur at musically natural points, for instance at the beginning of a musical gesture.

IV. Voice leading. The rules of classical voice leading still apply! Parallel fifths and octaves are still forbidden! You should be able to reduce your piece to something that looks roughly like a chorale, with good classical-style voice leading. Of course, in these reductions, the accompanimental voices will typically move more slowly than the soprano.

Watch out for parallels both between successive strong beats (beat 1, measure 1 and beat 1, measure 2) as well as immediately adjacent notes.

V. Writing the melody. In composing your melody, it is a good idea to try to use *motives*—short, recognizable rhythmic ideas that gives the melody character. You can then reuse your motives by transposing them. For example, in the first passage below, the first five notes of the right hand constitute the motive. Haydn repeats this motive exactly in mm. 3-4, then transposes it to the dominant. Note that he alters a bit—the original motive ended a fifth above its starting point, whereas the transposition ends a seventh above the starting point. He then repeats this transformed motive before concluding the phrase. (This may be a bit unusual—typically you would repeat the motive two or three times, not four times.) Often a phrase will have two or more motives. Usually they are developed sequentially rather than in alternation.

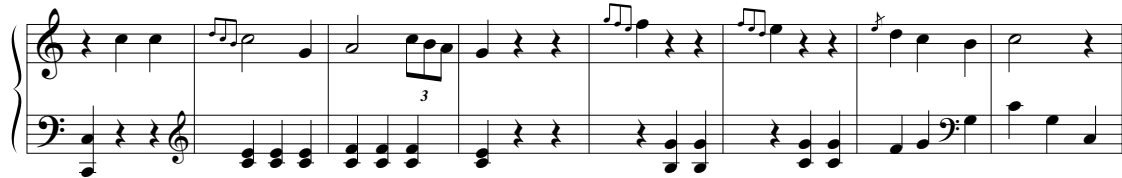
VI. Writing the accompaniment. The basic idea is to disguise the chorale-like background by decorating the rhythm. This process is called *rhythmic figuration*. There are a couple common figuration styles:

A. *Give the accompaniment the character of a second melody.* Make it interesting, though not as interesting as the soprano. Give it its own rhythmic motive(s). For instance, in the Haydn passage below, the passage in the second and third measures represents the accompanimental motive. It appears three times.



Note how the middle voice moves in parallel with the bass for the first part of the phrase, then in parallel with the soprano. You can think of the octave on the first beat as a single voice (rather than two voices) that appears in two octaves (“doubled at the octave”). This is very normal in Style 4, and does not constitute parallel octaves.

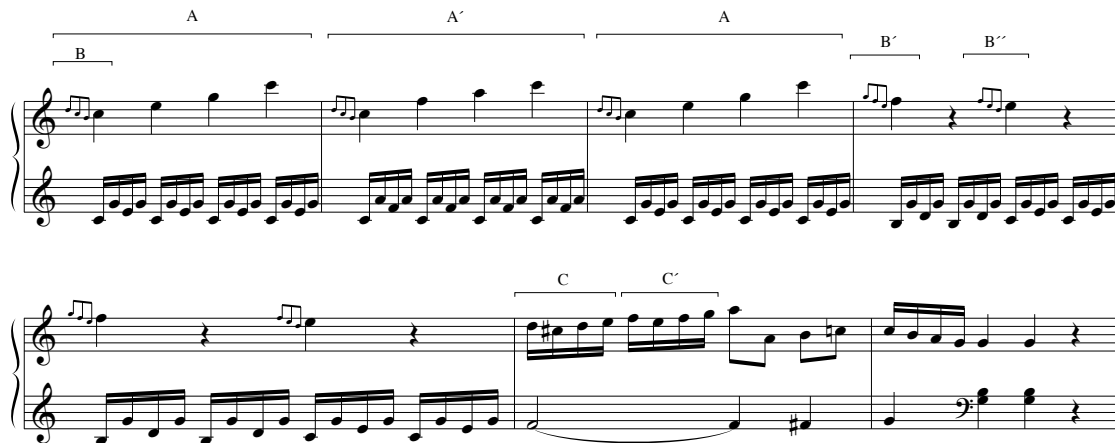
B. Use repeated chords. Judicious use of repeated chords can really give an accompaniment some zip, as in the following Haydn minuet phrase:



The first six measures contain one harmony per bar. In the seventh measure we get one harmony per beat, and then one harmony in the eighth measure. Note that the music loses a voice near the cadence. The melodic motives are more subtle here: the half-quarter rhythm of measure 2 is repeated in an embellished form in m. 3, as a half-note followed by a quarter filled with triplet eighths. (These triplet eighths recall the grace notes in m. 2). Furthermore, the right hand of m. 5 is transposed to produce m. 6.

We will discuss m. 3’s unusual IV_4^6 chord below.

C. Use an Alberti bass! It is very common for the accompaniment to arpeggiate the harmony in a systematic way. You’ve all heard this before:



It is important to understand that the left hand in mm. 1-5 is playing *three* separate musical voices, not a single line. The arpeggio sounds the bass voice first, then the alto, then the tenor, then the alto (BATA). Parallel fifths between these “virtual voices” are still forbidden!¹ Other common patterns include BTAT and BTA (in triplets).

¹ There are a very small number of examples where Mozart uses Alberti patterns that imply parallel fifths. Go figure!

Note Haydn's extensive use of motives! I've labeled three motives here, A, B, and C. (B is a part of C.) This guy knew what he was doing! The F $\sharp$ in measure 6 is part of a very common pattern. You can understand this as a chromatic passing tone, or as an "applied" dominant chord (V $\frac{6}{5}$) of G major.

VII. New chord progressions. Classical keyboard music makes use of a few new chord progressions.

(a) Pedal (or "neighboring") $\frac{6}{4}$ chords. In two of the examples on this handout, Haydn uses the progression I- IV $\frac{6}{4}$ -I, usually at the start of a phrase. Here the C stays fixed as a "pedal tone" in the bass. The chord is sometimes called a *neighboring* $\frac{6}{4}$ chord, since the F and the A act like neighbors to the G and the E. You can use this progression in classical-style keyboard music. You also commonly see V-I $\frac{6}{4}$ -V.

(b) Cadential $\frac{6}{4}$ chords. We have learned that I $\frac{6}{4}$ chords occur on strong beats at cadences. This is also true in classical-style keyboard music. However, in Style 3 vocal music the $\frac{6}{4}$ is always preceded by a ii (usually ii $\frac{6}{4}$) or a IV chord. In Style 4, I or I $\frac{6}{4}$ sometimes goes directly to the I $\frac{6}{4}$.

(c) Parallel thirds with the melody. Very often, classical music employs parallel thirds or sixths underneath the melody. Sometimes these parallel chords can be given a harmonic interpretation, but this is not always the case. You might, for example, see EG-FA-EG-DF-CE over a G in the bass.

(d) Cadences. Classical phrases very often end with a ii $\frac{6}{4}$ -I $\frac{6}{4}$ -V $\frac{7}{4}$ -I progression. **Use this very frequently!** (Like, 75% of the time or more!)

VIII. Sentences and periods. Classical phrases come in many different forms, limited only by the imagination of the composer. However, there are a few common phrase structures that you should know about. The two that people talk about are the *sentence* and the *period*. However, I think there's a third type that belongs in this category as well.

(a) The period. The period consists of two halves; usually each half is four measures long, but sometimes they are eight or even sixteen measures. The first half ends with a half cadence. The second half begins exactly the same way the first half does, but concludes with a full cadence. Therefore the phrase as a whole has an AA' structure. (The notation "A'" indicates a varied form of A.) A classic period:



(b) The sentence. The sentence begins with a two-bar idea (A). This two bar idea is then either repeated exactly, or stated in varied form (A'). The remainder of the phrase contains contrasting material that leads to a cadence. Therefore the phrase has an AA'B structure.

structure. Usually the A and A' sections are two measures long with the B phrase lasting four measures. However, sometimes the B section is longer.

The most common harmonic pattern for a sentence is A: I – V, A': V – I. However, there are many, many other possibilities.



(c) Think of a classical 8-bar phrase as having four 2-bar units, ABCD. The final two bars usually form a cadence, and hence will not typically contain much thematic material. Now if you want your phrase to have some sort of unity, it makes sense for two of the remaining three bars to have some thematic connection. In the period, the relationship is between the first and third units, giving ABA'D. In the period, it is between the first and second units giving AA'CD. Since you can't put an A' in the fourth position (since the D is reserved for generic cadential material), there's only one more possibility: ABB'D. For a good example, look at the first eight bars of the first example on your Haydn "rounded binary" handout, where bars 5-6 relate to bars 3-4. This phrase type has no name, but it makes perfectly good sense, and arises out of the same principles that give rise to the better-known period and sentence.



(d) The phrase extension. Classical composers were tricksters. They loved setting up expectations, only to thwart them! A common way to do this is to lead the listener to expect a strong cadence, and then to evade the strong cadence somehow, prolonging the music with a little extra material before arriving at the expected cadence. Sometimes one extends the phrase by repeating the material used to end the phrase.

Sometimes, one actually reaches the cadence, but extends the phrase anyway, with a little extra material. (See the Mozart G major sonata on the “sentences” handout.)



The Mozart excerpt above is a sentence, with the first two bars repeated almost exactly. The cadence in bar 8 is somewhat inconclusive, since it ends with the third scale degree (E) and uses the chromatic (incomplete) neighbor D#. We then get a varied repeat of bars 5-7 of the phrase, reaching a more conclusive cadence on the first scale degree in m. 12. A classic phrase extension!

Sometimes, instead of a phrase *extension*, the phrase will have a brief introductory passage; see, for example, the opening of the first Mozart sonata.

IX. Rounded binary form. Rounded binary form is the smallest complete form in the classical style. It consists of a first phrase (typically eight bars) which is immediately repeated; a contrasting middle section (often 2, 4, 6, or 8 bars), and a repeat of the first phrase (sometimes altered, and sometimes truncated to four bars), with the last two parts also being repeated:

||: A :|| ||: B A' :||

One can think of rounded binary form as a simple device that allows the composer to repeat a single phrase (the A) several times without boring the listener. For examples, consider Beethoven's *Ode to Joy*, the theme of the third movement of Mozart's Sonata in D major K284 (p. 54 in the Dover edition), the theme of the first movement of Mozart's Sonata in A major K331 (p. 118), the first 24 bars of the third movement of the same sonata (p. 126), the first 22 bars of the third movement of Mozart's B $\flat$ major sonata K570 (p. 246), or any of the Haydn examples on the Xeroxed collection I handed out (also available on blackboard).

The A section can be either a sentence or a period, or some other form entirely. It is very common for the A section to modulate: in major, the standard modulatory destination is the dominant key; in minor it is either the relative major or the minor dominant.

The B section provides a harmonic contrast from the A section. Often, the phrasing is reversed, so that the first measure of the B section begins with a V chord, rather than a I.² Sometimes it modulates to new keys. It typically comes to rest on a V chord in the tonic key, preparing for the return of the A section.

If the opening A section modulated to the dominant, the repeat of the A section will be altered to end in the tonic. If the opening A section is a period, the repeated A sometimes uses just the second half of the phrase.

A very common classical form combines two rounded binary forms, often called *minuet* (or *scherzo*) and *trio*. These are arranged in an ABA pattern so that you have minuet-trio-minuet. Usually, the repeats are omitted during the second performance of the minuet.

One also finds longer rounded binary forms in which the A and B are much larger than 8 measures. In fact, rounded binary form is the ancestor to sonata form (more about this next semester) which can be hundreds of measures long!

² Classical music is often arranged into two-bar unit, with a strong measure followed by a weak measure. (The technical term for this is “hypermeter”: a rhythmic structure in which measures are the units.) In the opening section of a piece, the I chord is often found on “strong” measures. The B section can create contrast by inverting this arrangement, placing the V chord on the strong measures instead.

1. **Allegro**

7

13

19

2. **SCHERZO (Allegro)**

6

12

18

*)

3

14

MENUETTO

Measures 1-14 of the Menuetto. The piece is in 3/4 time with a key signature of one sharp (F#). The first system (measures 1-6) features a treble staff with eighth-note triplets and trills, and a bass staff with a steady eighth-note accompaniment. The second system (measures 7-14) continues the melody with various fingerings and trills, while the bass staff maintains the accompaniment pattern.

Measures 15-22 of the Menuetto. The treble staff continues with eighth-note triplets and trills. The bass staff accompaniment remains consistent, providing a rhythmic foundation for the melody.

TRIO

Measures 23-30 of the Trio section. The treble staff features a new melody with trills and triplets. The bass staff accompaniment changes to a pattern of eighth notes and quarter notes, with a 2/4 time signature indicated at the beginning of the system.

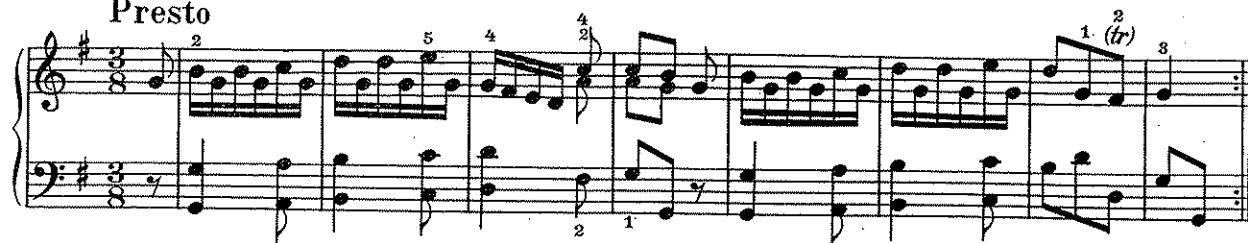
Measures 31-37 of the Trio section. The treble staff continues with a melodic line featuring trills and triplets. The bass staff accompaniment consists of eighth notes and quarter notes, maintaining the Trio's rhythmic feel.

Measures 38-44 of the Trio section. The treble staff concludes the Trio melody with trills and triplets. The bass staff accompaniment continues with eighth notes and quarter notes. The section ends with a double bar line.

Menuetto da Capo

4

*) FINALE
Presto



*) vgl. Sonata No.5, erster Satz (siehe Vorwort).
cf. Sonata No.5, First Movement (see Preface).

da Capo al Fine

5

20

TRIO

25

Musical notation for measures 25-30. The key signature is one sharp (F#). The melody in the right hand features triplet eighth notes in measures 25, 26, 28, and 29, and a triplet quarter note in measure 30. The bass line consists of chords and single notes. A double bar line with repeat dots is at the end of measure 30. A footnote marker (*) is present in measure 30, with a corresponding '2 4' below the bass line.

31

Musical notation for measures 31-36. The key signature is one sharp (F#). The melody in the right hand includes a triplet quarter note in measure 31, followed by chords with fingerings 5, 4/2, 4/2, 4/1, 3/2, and 3 in measures 32-35. Measure 36 has a triplet eighth note. A double bar line with repeat dots is at the end of measure 35. The bass line includes chords and single notes with fingerings 3, 1, and 2.

37

Musical notation for measures 37-42. The key signature is one sharp (F#). The melody in the right hand features triplet eighth notes in measures 37, 39, 40, 41, and 42, and a triplet quarter note in measure 40. The bass line consists of chords and single notes. A double bar line with repeat dots is at the end of measure 42.

43

Musical notation for measures 43-48. The key signature is one sharp (F#). The melody in the right hand features triplet eighth notes in measures 43, 44, 45, and 48, and a triplet quarter note in measure 46. The bass line consists of chords and single notes. A double bar line with repeat dots is at the end of measure 48.

49

Musical notation for measures 49-54. The key signature is one sharp (F#). The melody in the right hand includes a triplet quarter note in measure 49, followed by chords with fingerings 1, 3, 3, 1, and 2 in measures 50-53. Measure 54 has a triplet eighth note. The bass line includes chords and single notes. A double bar line with repeat dots is at the end of measure 54.

Sentences

BEETHOVEN, Op. 2, No. 3

Allegro con brio

3

First system of the musical score for Beethoven's Op. 2, No. 3. It consists of two staves, treble and bass. The treble staff begins with a piano (*p*) dynamic and features a series of chords and eighth-note patterns. The bass staff also starts with a piano (*p*) dynamic and includes a measure with a boxed number '5'. The system concludes with a forte (*ff*) dynamic and a series of descending eighth notes. Fingering numbers (1-5) are indicated above various notes.

W. A. Mozart.
Köchel-Verz. Nr. 283.

Allegro.

5.

First system of the musical score for Mozart's Köchel-Verz. Nr. 283. It consists of two staves, treble and bass, in a key with one sharp (F#). The treble staff begins with a piano (*p*) dynamic and features a series of eighth-note patterns. The bass staff also starts with a piano (*p*) dynamic and includes a measure with a boxed number '5'. The system concludes with a forte (*f*) dynamic and a series of descending eighth notes. Fingering numbers (1-5) are indicated above various notes.

Periods

W. A. Mozart.
Köchel-Verz. Nr. 331.

11. Andante grazioso.

Handwritten musical score for the first system of 'Andante grazioso'. It features a treble and bass staff in G major (two sharps) and 6/8 time. The treble staff begins with a piano (*p*) dynamic. The bass staff has a steady eighth-note accompaniment. The system contains six measures.

Handwritten musical score for the second system of 'Andante grazioso'. It continues the piece with a treble and bass staff. The treble staff has a forte (*f*) dynamic in the first measure, followed by a piano (*p*) dynamic. The bass staff continues with eighth notes. The system ends with a repeat sign.

Mozart

Allegretto grazioso.

Handwritten musical score for the first system of 'Allegretto grazioso'. It is in G minor (two flats) and 3/4 time. The treble staff starts with a piano (*p*) dynamic. The bass staff has a steady eighth-note accompaniment. The system contains six measures.

Handwritten musical score for the second system of 'Allegretto grazioso'. It continues the piece with a treble and bass staff. The treble staff has a forte (*f*) dynamic. The bass staff continues with eighth notes. The system ends with a repeat sign.

Mozart

Rondo.
Allegro.

Handwritten musical score for the first system of 'Rondo Allegro'. It is in G minor and 3/4 time. The treble staff starts with a piano (*p*) dynamic. The bass staff has a steady eighth-note accompaniment. The system contains six measures.

Handwritten musical score for the second system of 'Rondo Allegro'. It continues the piece with a treble and bass staff. The treble staff has a forte (*f*) dynamic. The bass staff continues with eighth notes. The system ends with a repeat sign.

①

Allegro con brio

A musical score for the song 'The Rose Tree'. It consists of two staves. The top staff is in treble clef and contains a melody with various ornaments (dots) and fingerings (5, 3, 3, 2, 3, 4). The bottom staff is in bass clef and contains a bass line with some rests and a final note marked with a '5'.

②

Andante con Variazioni.

12. *Andante con variazioni.*

p *cresc.* *sf* *p* *cresc.* [5]

Mozart

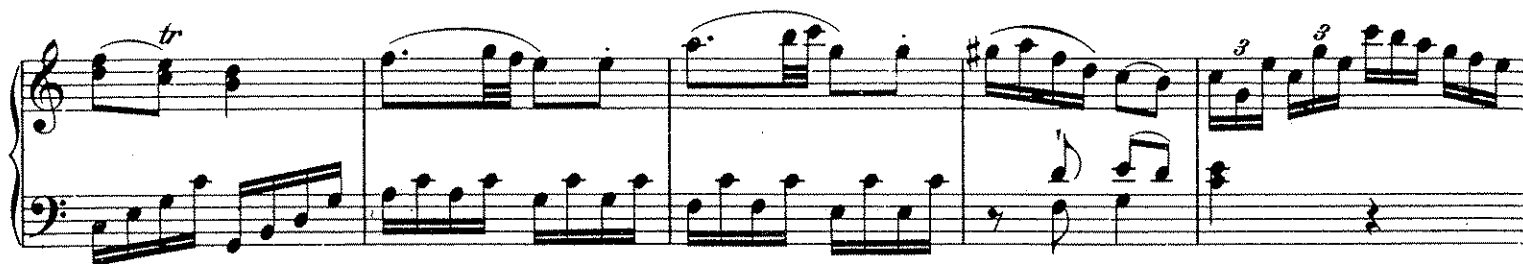
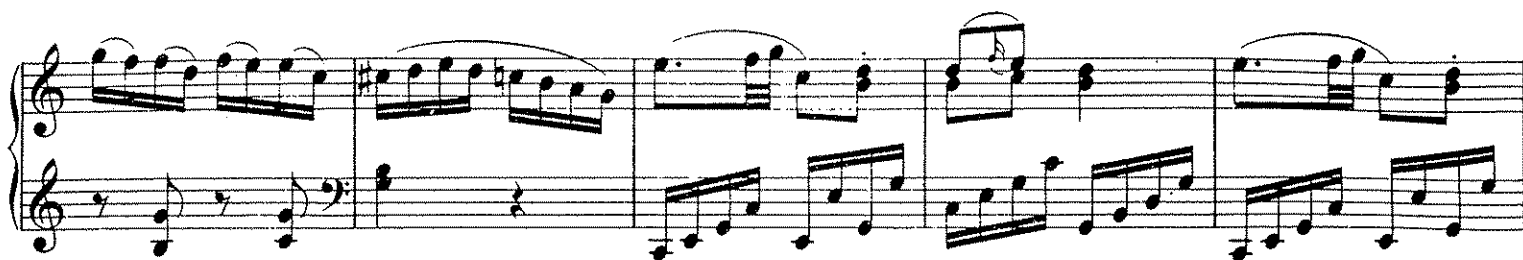
Rondo.
Allegretto grazioso.

3



Allegretto.

4



MUSIC 105

Dmitri Tymoczko
Handout 19 (2010)

HOW TO WRITE A BAD¹ CLASSICAL PHRASE

Periods are easier than sentences, since they involve less writing. Measures 1-2 can usually be repeated to form measures 5-6. However, the same basic instructions should work for sentences.²

Step 1. Choose a relatively simple meter, and a moderately fast tempo.

Example: let's choose 3/4 with at quarter = 152.

The reason for this is that certain meters and tempi (for instance, 6/4 with a tempo of quarter = 50) require you to write a lot of notes. Writing a lot of notes is harder than writing just a few notes. Why not make things easy for yourself?

Step 2. Choose a straightforward, clear chord progression for the first 4 measures. Since this is a period, we want the first four-measure phrase to end on V.

Example: let's use I-V-I-V, with each chord lasting for one measure.

Step 3. Without worrying about the accompaniment, try to find a simple melody that goes with these chords. Make sure every note in the melody is either part of the chord, or is a recognizable non-harmonic tone such as a passing or neighboring tone. Do not allow any “rogue notes” – i.e. notes that do not belong to the chord, and yet do not belong to an accepted non-harmonic tone category.



Since we are writing the first phrase of a period, we want our melody to end on scale degree 2 or 7. That way, when the second phrase ends on scale degree 1, it will sound like it has “answered” the “question” posed by the first half.

¹ It really is helpful to think of yourself as trying to write bad *but typical* classical music. The reason is that it's easy to get overwhelmed by the greatness of great classical music, and to feel that you have to write something overwhelmingly great in order for it to count as “classical music” at all. This isn't true. A lot of classical music is banal, trite, and boring. *This is the music that we're trying to write*, in part because it will enable to see why great classical music is great. By writing boring *but typical* classical music, we can learn the conventions of the style. By learning the conventions of the style, we can see how the great composers deviated from them.

Some of you may be ambitious, and may aspire to write great music yourselves. This is a good thing. But it is unrealistic to expect yourself to write good classical music after only 3 or 4 months of study. Classical music is like Latin – a language that is no longer spoken in everyday life. It will take years for you to become fluent in this language. In the meantime, don't let your ambitions to be great prevent you from being adequate!

² Including the suggestion that you write the melody for measures 1-4 before 5-8.

Step 4. Now, thinking about both melody and harmony, try to figure out how to change the end of the first 4-measure phrase so that it concludes on the tonic. You may not need to change the melody very much. However, some melodies will require more extensive alterations.

In general, you want measures 5-6 to be the same as 1-2. You don't need to vary these measures to "keep things interesting." In a period, the variation usually comes at the end of a phrase, not the beginning.



Notice that our whole melody contains only 20 notes, and that 8 of them are repeats! That means it really isn't *that* hard to write a bad (*but typical!*) classical phrase.

Step 5. Now that we have our melody and chord-progression, we need to add accompanimental voices. Start with very simple harmonies, trying to keep the voice-leading as clear as possible, and avoiding parallels. You may find yourself changing inversions, or even adding a chord here and there, in order to produce a smooth accompaniment.



Notice here that we've put the V chord in measure 2 into second inversion. In measure 3, we have two different inversions of the I chord. This is because we're trying to write an independent bass line that is somewhat interesting in its own right.

We've also shifted the melody up by an octave. This is because the accompanimental chords sound muddy when they're in the octave below middle C. Many beginning students make the mistake of writing music that is too low. In general this is NOT stylistic: classical music tends to have a relatively high range. (Another musical term for a registral range is *tessitura*, so that we could say "classical music has a high tessitura.")

Now at this point, many of you may want to stop. You've written a boring but typical classical period, with a boring but acceptable melody, and boring but correct accompaniment. You deserve a boring (*but decent!*) grade. Some of you, however, may want to continue on to ...

Step 6. (optional) The final stage is to give our phrase a little more zip by adding some *rhythmic figuration* to the accompaniment.

- In measures 1-2, I have used a standard classical "Alberti bass" pattern to enliven the harmony.
- In measure 3, I added a passing chord (D-F in the left hand) on beat 2.
- In measure 4, I composed an arpeggio to keep the rhythmic energy from fading.
- Measures 5-6 are exactly the same as measures 1-2.
- in measure 7, I added a D to the accompaniment (4th eighth note) to create a stronger sense of cadence.
- In measure 8, I repeated the arpeggio idea from measure 4.



Do not undercut your earlier work! In particular, don't obscure the basic harmonic progression (Step 4) or the voice-leading that you sketched in Step 5. Figuration is the icing on the cake. Don't let it distract from the cake itself.

You now have a perfectly normal, typical classical phrase. It's boring, but correct. If you were training to be a great classical composer, you would write hundreds and hundreds of these phrases, knitting them together to form pieces of music. As you wrote more and more, your phrases would gradually get more and more interesting. Eventually, your music would be neither boring nor typical, but would instead become great.

KEY RELATIONS

Theorists agree that tonal music tends to modulate between keys that are closely related. But which keys are these, and how can we visualize the relationships among them? These questions turn out to be surprisingly difficult and interesting.

I. Shared notes, key distances, and voice leading. In theory, one could imagine musicians deciding that the keys of C major and D $\flat$ major were “close”—after all, the note C is right next to the note D $\flat$ on the keyboard. And in fact, there are some situations where this notion of “closeness” is very useful. Consider a pop/rock context where you might have just a single singer.

Q: if you want to transpose a passage of music into a new key, what transposition makes the most sense?

A: transposition by one or two semitones, so you don’t lead the singer out of his/her vocal range!

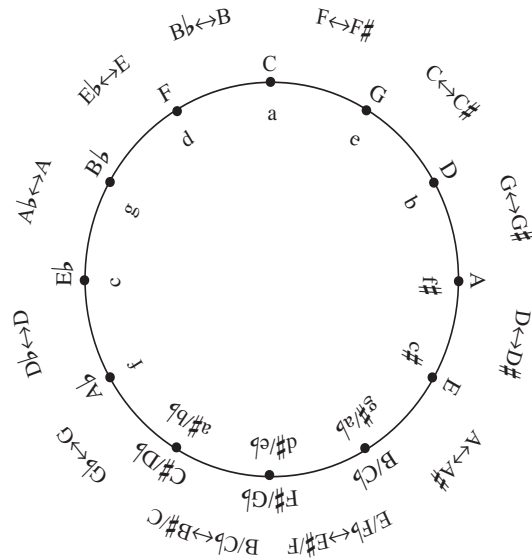
And in fact, pop music often features what’s called the “pump-up modulation,” where the music jumps up by a semitone or major second (cf. “Good Day Sunshine” by the Beatles; “Surrender” by Cheap Trick, and many other examples).

But this is not what happens in classical music: C-major pieces rarely modulate to D $\flat$ major, and these two keys are thought to be quite far apart. This is because classical composers use an alternate notion of distance whereby the keys of C major and G major are said to be maximally close. Note that there are two possible explanations for this closeness. The first is that the note G is a fifth above the note C, with the fifth being the most consonant interval besides the octave. The second is that the *scale* of G major shares six of its seven notes with the scale of C major, differing only by the single-semitone shift F $\rightarrow$ F $\sharp$. Thus the first explanation emphasizes the acoustic relation among the tonic *notes*; the second emphasizes the common-tone or voice-leading relationships among *scales*. Choosing between these two explanations turns out to be surprisingly difficult.

NB: be sure you are clear on the distinction between a *scale* and a *key*. Keys are more general than scales! A major key uses only one scale, but a minor key uses three scales (melodic, natural, and harmonic minor). Thus there is no *key* of “A melodic minor”; the term “A melodic minor” refers to a *scale* (a collection of notes that can be used to build melodies and chords, as explained in earlier handouts) rather than a *key*. Nevertheless, it is reasonable to try to use scale relations to determine a set of distances among keys.

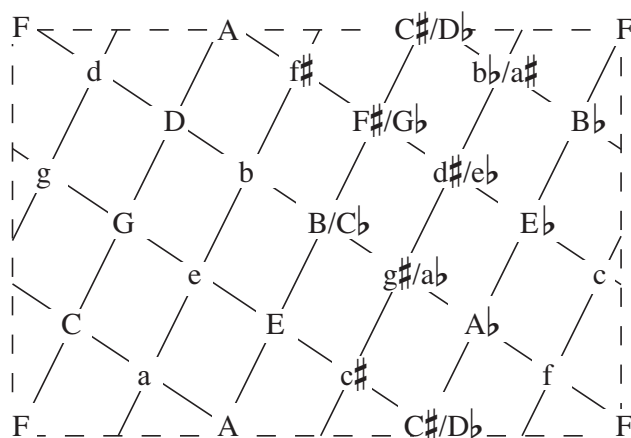
II. Simple fifth-based, relative major/minor tonality. Diatonic scales related by perfect fifth share six of their seven notes: one can move from C major to G major by

changing just one note, F, by one semitone, to F $\sharp$. Similarly, the major scale and the natural minor scale share all of their notes. For this reason, one can modulate between a key and its relative major and minor with very little effort—for instance, from C major to A minor, or from D minor to F major. The following circular diagram (originating with Kellner in 1737) provides an elegant representation of these relationships: major keys are on the outside of the circle; minor keys are on the inside. To move between a key and its relative major or minor, move along the circle's radius (i.e. from inside the circle to outside, or vice versa). To move by fifth, move along the circumference of the circle.



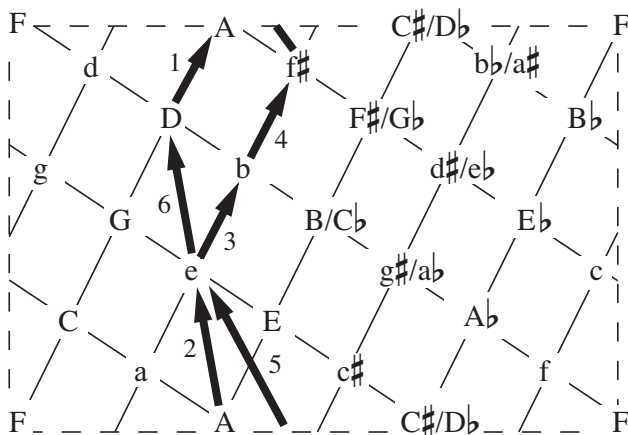
This circle does a pretty good job capturing the Baroque (1600-1750) conception of tonality: a Baroque piece tends to occupy a small region on this map, centered around the tonic key. So if a piece begins in G major, it might move to C major, D major, A minor, E minor, and B minor. But it would not typically move to keys that are much farther away on the circle. It is a good exercise to take a Bach chorale and follow its progress on the map.

III. The parallel relationship. The circle doesn't tell the whole story, however. For instance, one might think that C major and C minor are closely related—not so much because their scales share lots of notes, but because they share the same *tonic*. We therefore need to replace the simple circular diagram with a more complicated geometrical model, in which *parallel* major and minor keys (like C major and C minor) are also close together.



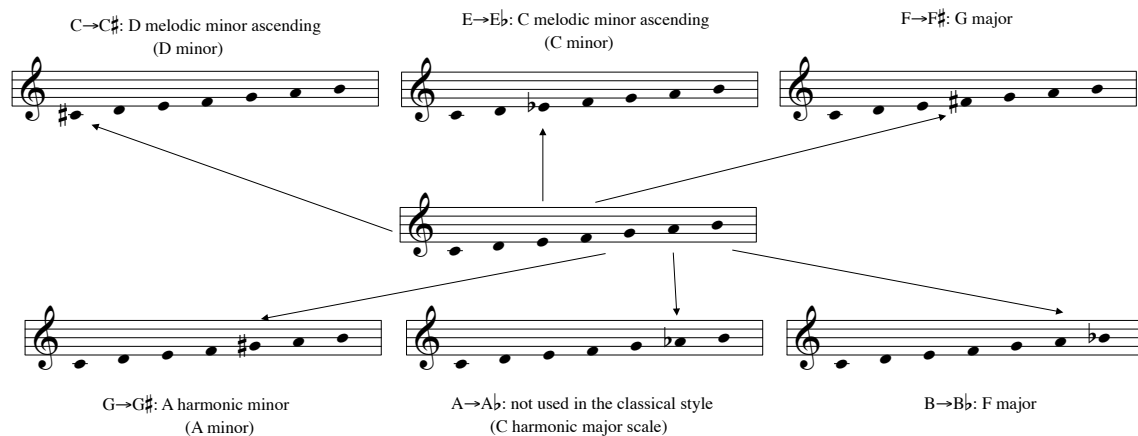
The above diagram can be useful here. (It was invented, more or less, by F. G. Vial in 1767.) Major keys are represented by capital letters, minor keys by small letters. You should imagine that the top edge is glued to the bottom edge; and the left edge is glued to the right. (Just like the old video game “Asteroids.”) Moving along the circle of fifths corresponds to moving along the SW/NE diagonal lines. Thus we can move diagonally from F to C to G to D to A; we then jump from the top of the diagram to the bottom (remember, they’re glued together!), proceeding onward from A to E to B, etc. Meanwhile, when we move along the SE/NW diagonal, we move between major and minor keys, alternating between the parallel and relative relationship. For example: from E major we move NW to E minor, its parallel minor, and then to G major, the relative major of E minor, then G minor (parallel of G major), then (wrapping around from left to right!) to B $\flat$ major, etc.

It is useful to plot pieces of music as they move around on this graph. Below, I’ve charted the first six modulations found in the Bach D major prelude from Book I of the *Well Tempered Clavier*. These modulations begin and end in D major, passing through a series of intermediate keys. See if you can extract from this geometrical depiction some useful information about how Bach moves from key to key. Does he like any particular patterns? Are there any differences between the beginning of the journey and the end?



IV. Voice leading between scales. We can actually go a little deeper than this. on the preceding graphs, the keys of D minor and E minor are equidistant from C major. However, we cannot establish a minor key without introducing a leading tone. From this point of view, it's reasonable to wonder whether C major and D minor might be closer together than C major and E minor are: to move from C major to the V^7 of D minor you need add only one accidental ($C^\sharp$) whereas to get to the V^7 of E minor you need two ($D^\sharp$ and $F^\sharp$). Moreover, the three forms of the D minor scale on average share more notes with those of C major (5.7) than do the three scales of E minor (5).

Below I've listed all the familiar scales that be obtained by changing one note of the C major scale by one semitone. Note that D melodic minor occurs on this list, but not E melodic or harmonic minor.

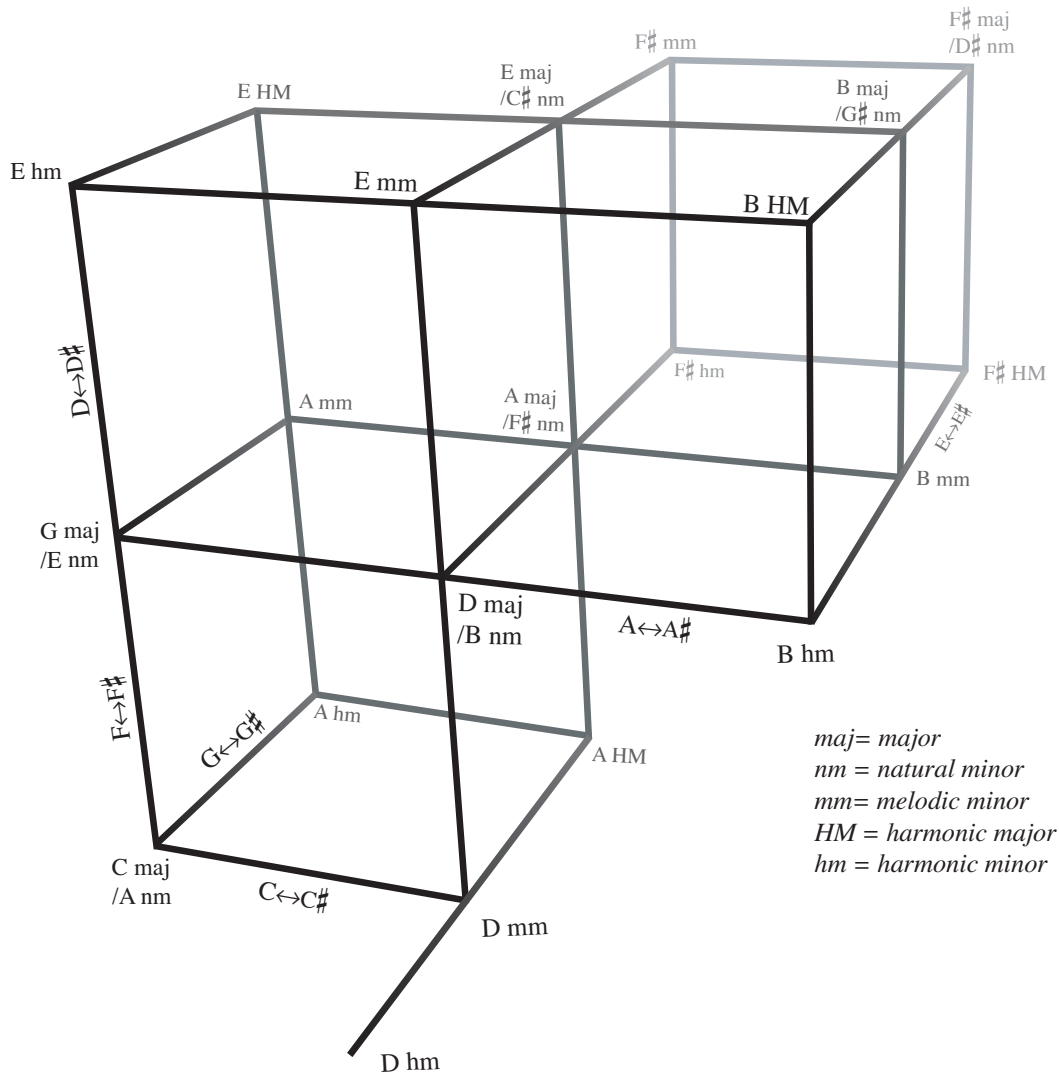


This leads to the thought that two keys are close if their associated scales can be linked by small voice leading motions. Furthermore, this might lead us to wonder how we might visualize the single-semitone voice leadings between all the familiar seven note scales. It turns out that it takes three dimensions to do it! The diagram on the next page illustrates. It's very hard to draw the full diagram on a two-dimensional piece of paper, so I've shown only an excerpt of the full structure.

It is worth taking a little time to explore this diagram. The circle of fifths zigzags through the figure, starting at the lower left with C major, moving up to G major, right to D major, and into the paper to A major. Wound around this zigzag of fifths is a sequence of nondiatonic scales: D mm → A HM → A hm → A mm → E mm → etc. Alternatively, one can understand the graph as a series of cubes stacked next to each other: the second cube is on top of the first cube; the third is to the right of the second; the fourth is behind the third, etc. Q: how are the individual cubes related to each other musically? How are they related geometrically? Can you complete the structure? (Again, it can be interesting to use this diagram to track a piece's movements from scale to scale.)

Now it turns out that distances on this graph more closely track the modulation frequencies in actual pieces of music, than do distances on our earlier graphs: it seems that these sorts of subtle voice-leading relationships between scales do indeed capture something about "key distance" as understood by baroque and classical composers! Of

course the details are subtle, since the graph represents distances between *scales*, which need to be converted into distances between keys. For more information about how to do this, consider taking my advanced theory class!



V. Heirarchical self similarity. The deep moral is that tonal music involves very similar procedures on two different temporal levels. On the *local* level, a tonal piece moves from chord to chord, linking the notes of these chords by smooth voice leading. Over a larger span of time a tonal piece modulates from key to key—typically by way of subtle shifts that take you from one scale to another closely related scale. For example, we can modulate from C major to C minor by changing the E $\sharp$ in the C major scale to an E $\flat$, forming a C melodic minor scale. These scale-to-scale transformations as involve something very similar to the efficient voice leading we find on the chord-to-chord level. In both cases, it is a matter of keeping as many common tones as one can, and moving the remaining notes by the smallest possible distance.

In other words: tonal music is using the same processes (efficient voice leading between a small number of similar musical objects) on two different time scales. On the chord-to-chord level, we call this “voice leading.” On the larger, scale-to-scale level, we call this “modulation.”

Examine the following excerpt from a Bach chorale. The smooth voice leading on the chord-by-chord level should by now be familiar to you. But note how the succession of scales (shown on the bottom staff) suggests a very similar sort of voice leading, one that happens more slowly in time.

